The redistributive effects of sectoral shocks *

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Abstract

This paper investigates the redistributive impacts of sectoral shocks. We develop a new model combining features from both the heterogeneous agents and input-output literature, which is amenable to analyse how idiosyncratic shocks propagation redistribute wealth and income within sectors. We find that key sectors in shock transmission can be identified using the sectoral influence centrality measure. Our model also suggests that negative supply shocks generate positive redistribution effects for the supplying sector, whereas receiving industries suffer distributional costs. We extend our analysis by introducing fiscal policies and show that while labor tax shocks trigger homogeneous responses for inequalities across sectors, capital tax shocks generate very heterogeneous responses with substantial distributional costs.

Keywords: sectoral shocks; propagation; heterogeneous agents; production networks; redistribution.

JEL: D31, D57, E21, E22, E23, E25, E30, E32, O41

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1 Introduction

The distributional implications of business cycles and economic policies are of growing interest for macro-economists. At the household level, wages represent the major source of personal income for most US citizens (almost 60% of the total income of US citizens consisted of wages and salaries in 2019).[1] This observation suggests that households' revenues are highly contingent on the activity of the industry in which the households are working. Figure 1 shows how consumers' earnings are subject to sectoral business cycles, with strong heterogeneity in wage levels as well as wage inflation and trends. This contingency between personal income and sectoral trends can be a forceful vector of intra-industry and across-industry wealth redistribution for households.

In this paper, we investigate the distributional impacts of sectoral shocks, using as a starting point the strong heterogeneity in earnings across industries. To tackle this issue, we develop a heterogeneous-agents model in continuous time featuring an Input-Output architecture for production (HACT-IO). This model bridges the gap between two branches of the literature whose aim is to model heterogeneity more accurately, either from the household side (building on the heterogeneous-agents literature) or from the sectoral side (building on the production networks literature). In particular, our model's unique feature is to explicitly associate each household with a specific sector by which it is employed. While contemporaneous works from Schaab and Tan [2024] also seek to dissect household-sector linkages through the lens of a "HANK-IO" model, our approach differs by classifying households according to their employment industry rather than their position in the income distribution. Hence, households' earnings are directly linked to sectoral business cycles, allowing for idiosyncratic shocks to propagate through sector-specific income channels. After disentangling the aggregate and sectoral dynamics in our HACT-IO economy, we study how sectoral shocks can generate distributional effects inside and across industries. We characterize how to identify key sectors in shock transmission, as these industries might require special attention from policymakers.

Over the last decade, the agenda of macroeconomics research have extensively addressed the question of heterogeneity. Heterogeneity is an inherent feature of any real economy and is often disproportionately hard to incorporate in macroeconomics model, giving rise to additional layers of complexity. The most recent contributions have documented two types of heterogeneity. On the household side, the burgeoning heterogeneous-agents literature has made substantial advances in understanding how the heterogeneity

[1] SOurce: FRED database.

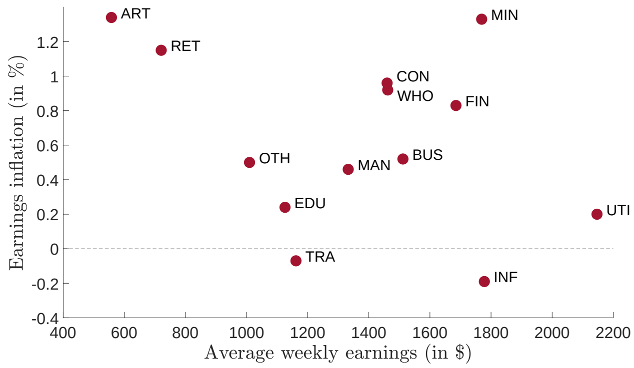


Figure 1: Employment and average weekly earnings by industry, (February 2024, source: BLS)

in consumers' wealth and income can affect macroeconomic dynamics as well as monetary and fiscal policies transmission. On the production side, the Input-Output literature has considerably enriched the perception of propagation mechanisms of economic shocks along supply chains. Incorporating any of these features in a model bears heavy costs in terms of computations and resolution. By combining both heterogeneity, our HACT-IO model meets this challenge and takes a major step towards enriching macroeconomic models and drawing a more accurate portrait of macroeconomic dynamics. However, we restrict ourselves to a 2-digit NAICS level of disaggregation, considering a 14-sectors version of the US economy for computational purpose.

We obtain three main findings from our quantitative analysis. First, we characterize how to identify key sectors in shock propagation, especially for distributional implications. In particular, we classify sectors according to two types of centrality measures, *sectoral influence* and *Bonacich centrality*, and show that the former best captures the importance of an industry in an economy. This result is rationalised by the analysis of the sectoral and aggregate dynamics following idiosyncratic shocks in our HACT-IO model. Most importantly, this result suggests that final demand for goods strongly drives redistribution effects.

Our second result addresses the redistributive impacts of sectoral shocks. By using projection methods, we are able to compute the wealth and income distributions of households at a sectoral level. We show that following a negative supply shock in a specific sector, downstream industries experience increase in inequalities and impoverishment on the short and medium run. Conversely, we show that the supplying sector (which is primarily hit by the supply shock) experiences an enrichment, with an upward shift of the wealth distribution and a decline in wealth inequalities. Above all, we find that key sectors for redistribution can also be identified using the sectoral influence centrality measure.

Our last result concerns fiscal policy. We extend the model by introducing a government which finances public expenditures with tax revenues. The tax authority has two instruments at its disposal to generate revenues: a labor income tax and a capital tax. Our results suggest that following a tax cut (for any of the tax), the sectoral outputs fluctuate according to a Public-to-private ratio which embeds how important is an industry for public consumption as compared to private consumption. We find that labor income tax cut generate positive wealth redistribution for all sectors. Conversely, we show that capital tax cuts trigger diverging inequality dynamics across sectors, with positive redistribution for industries with low Public-to-private ratios and high distributional costs for the others. These findings highlight potential difficulties for policymakers regarding redistribution.

Related literature. This paper contributes to three main strands of the literature on business cycles. Firstly, our paper belongs to the literature on input-output networks dating back to Leontief [1942], who provided the first analysis of the sectoral disaggregation of the US economy. Later on, Hulten [1978] defined a conceptual framework to understand the aggregate impacts of sectoral shocks. These works formed the basis of the first generation of multi-sector business cycles that emerged with Long and Plosser [1983], Horvath [1998, 2000] and Dupor [1999]. While partly abandoned until the last decade, the availability of large data sets and the improvements of computing powers and estimation methods led to a renewal in the interest of input-output models. Among others, works from Foerster et al. [2011], Acemoglu et al. [2012], Carvalho and Gabaix [2013], Acemoglu et al. [2015] and Carvalho et al. [2021] brought new insights on the role of downstream and upstream transmission patterns. In particular, these papers show how idiosyncratic shocks can induce sizable fluctuations in the aggregate economy when propagating through input-output linkages. Important contributions from Baqaee and Farhi [2018, 2019, 2020] also dissect the mechanisms at play in shock transmission from a theoretical perspective. Our paper adds to this literature by considering sectoral shock propagation in a full-fledged model where both production and demand side are disaggregated. We provide new insights on the channels of shock transmission by incorporating capital in our model, a feature which is sorely missing in most of production networks research.[2] Our model is amenable to study the propagation of shocks through the lens of sectoral aggregates, with the additional considerations on household wealth and productivity distribution.

Our paper is also related to the body of research examining the linkages between households and sectors. These linkages can be studied through the lens of either expenditures or earnings heterogeneity. Regarding expenditures heterogeneity, many empirical studies have already outlined the strong dependency of households' consumption basket on income (see Jaravel, 2019, 2021, Cravino et al., 2020, Comin et al., 2021, Andersen et al., 2022). Our paper does not address this issue (as we assume identical preferences across households) but rather tackles the implications of earnings heterogeneity. The main contributions in this direction are Clayton et al. [2019], who show that prices are more rigid in sectors employing college-educated households and Schaab and Tan [2024] who dissect how sectoral features are strongly correlated with the households' earnings. A stark difference with these works and ours is the explicit linkage we impose to each household with its employing sector in the model. By assuming that each household is assigned a single sector for which it is working, the earnings of the agents become strongly dependent on their employing industry's business cycles. This feature allows us to study how shock propagation through production networks translates into earnings shocks for the households and affects intra-sectoral inequalities.

Finally, our paper also contributes to the fledgling literature studying household heterogeneity in business cycles.[3] In particular, recent seminal contributions such as McKay and Reis [2016], Kaplan et al. [2018] or Auclert [2019] provide new insights for policy-making in heterogeneous-agents environments. From a methodological standpoint, various techniques can be drawn from the heterogeneous macroeconomic literature (such as Ahn et al., 2017 or Auclert et al., 2021) to solve dynamic systems. As our paper investigates distributional issues, we rely on the approach from Achdou et al. [2022] using the finite difference method for solving our heterogeneous-agent model. This projection method is well-suited to track the evolution of wealth and income distributions. We contribute to this literature by enriching typical heterogeneous agents models with the input-output architecture set out in production networks models. To our knowledge, the only existing HA-IO model is the HANK-IO framework from Schaab and Tan [2024]. Our paper differs from their specification by explicitly associating household types with their

[2] A notable exception is the trailblazing paper of vom Lehn and Winberry [2022].

[3] The first heterogeneous agents models can be traced back to Huggett [1993], Aiyagari [1994] and Krusell et al. [1998]

employing sector. This novel specification delivers a highly detailed representation of the economy, with a wealth distribution for each sector considered. This feature provides a framework for inspecting how sectoral disturbances may generate distributional costs across industries.

The remainder of the paper is organized as follows. In Section 2 we develop a general framework of production networks with heterogeneous-agents. Section 3 presents our solution method for the HACT-IO model. Section 4 dissects the dynamics of shock propagation with respect to different types of sectoral disturbances. Section 5 depicts the distributional effects of sectoral shocks. Finally, Section 6 extends our results to environments with fiscal tools.

2 The model

In this section, we develop a multi-sectoral model combining ingredients from the production networks framework as well as the continuous-time heterogeneous-agent literature. Time is continuous with periods $t \geq 0$ and horizon is infinite. Our economy differs from standard models by explicitly assuming household-sector linkages in the sense that each household works for one, and only one single sector. Consequently, households' earnings directly stem from their associated sectors' cycles. This characteristic, as well as time continuity, are our main departures from the "HANK-IO" model of Schaab and Tan [2024]. A schematic representation of the model is provided at the end of the section, in Figure 2

There are N production sectors in the economy, and thus N different types of households. Sectors exhibit input-output linkages such that all variables in the model are inter-dependent. Households differ in their wealth, their idiosyncratic risks and their employing sector. These three sources of heterogeneity determine the households' differences in consumption and savings policies.

2.1 Production network

The production side of the economy is embodied by N sectors. To produce, sectors use capital, intermediate inputs from the other sectors and labour force from the population. Sectoral capital is idiosyncratic, such that capital in sector j is produced by sector j and possessed by households working in sector j. The production technology is Cobb-Douglas with constant returns to scale with respect to these three factors, such that for all sector

j and time t :

$$Y_{j,t} = \xi_j Z_{j,t}\, K_{j,t}^{\alpha_j}\, L_{j,t}^{\beta_j}\, M_{j,t}^{1-\alpha_j-\beta_j} \tag{1}$$

where $Y_{j,t}$ is the output, $Z_{j,t}$ is a Hicks-neutral sectoral technology shifter, $K_{j,t}$ is the capital stock, $L_{j,t}$ is the labor use, $M_{j,t}$ is the basket of intermediate inputs use from all sectors and $\xi_j = (\alpha_j^{\alpha_j} \beta_j^{\beta_j} (1-\alpha_j-\beta_j)^{(1-\alpha_j-\beta_j)\sigma_j/(\sigma_j-1)})^{-1}$ is a normalization constant.

A production network emerges in this economy when industries are interconnected via multiple buyer-supplier relationships. Specifically, each industry j is a node in the network that is connected to other suppliers $i = 1, .., N$ by the purchase of intermediate goods. The total intermediate input demand of firm j takes the form of a CES technology as follows:

$$M_{j,t} = \left(\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} M_{ji,t}^{\frac{\sigma_j-1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j-1}}, \tag{2}$$

where $M_{ji,t}$ denotes the quantity of goods from sector i used as an input, the parameter $\sigma_j > 0$ is the elasticity of substitution across inputs for industry j, and $\gamma_{ji} \in [0,1]$ is the equilibrium intermediate expenditure share of sector j for input i. This specification imposes the following condition for constant returns to scale: $\sum_{i=1}^{N} \gamma_{ji} = 1 - \alpha_j - \beta_j$ for $j = 1, 2...N$.

Sectors maximize their nominal profits, which is defined as the total revenues minus factor expenses:

$$\Pi_{j,t} = P_{j,t}Y_{j,t} - w_{j,t}L_{j,t} - (r_{j,t} + \delta)K_{j,t} - \sum_{i} P_{i,t}M_{ji,t} \tag{3}$$

where $P_{j,t}$ is the nominal price of good j, $w_{j,t}$ denotes the nominal wage in sector j, $r_{j,t}$ nominal interest rate on capital $K_{j,t}$ of sector j and δ is the rate of capital depreciation.

Given the prices, the first order conditions yield the following set of equations on intermediate inputs, capital and labor, respectively:

$$M_{ji,t} = (1-\alpha_j-\beta_j)\gamma_{ji}P_{i,t}^{-\sigma_j} \left(\sum_{k=1}^{N} \gamma_{jk} P_{k,t}^{1-\sigma_j} \right)^{-1} P_{j,t}Y_{j,t} \tag{4}$$

$$r_{j,t} = \alpha_j \frac{P_{j,t}Y_{j,t}}{K_{j,t}} - \delta \tag{5}$$

$$w_{j,t} = \beta_j \frac{P_{j,t} Y_{j,t}}{L_{j,t}} \tag{6}$$

Defining the sectoral price index of intermediate inputs for sector j, $\tilde{P}_{j,t} := \left(\sum_{k=1}^{N} \gamma_{jk} P_{k,t}^{1-\sigma_j} \right)^{\frac{1}{1-\sigma_j}}$ allows us to reinterpret Equation 4 in terms of factor prices:

$$M_{ji,t} = (1 - \alpha_j - \beta_j) \gamma_{ji} \frac{P_{j,t} Y_{j,t}}{\tilde{P}_{j,t}^{1-\sigma_j} P_{i,t}^{\sigma_j}} \tag{7}$$

2.2 The households

There are N types of households depending on their employing sector (*i.e.* each household is assigned a single sector for which it is working). We denote with index j the variables referring to the households working for sector j. Labor supply is inelastic in each sector, such that : $L_{j,t} = \tilde{L}_j, \forall j$. Normalizing the mass of households across sectors leads to the condition : $\sum_j \tilde{L}_j = 1$.

Within a same sector, households are heterogeneous in wealth and productivity. Capital is possessed by the households, who can trade a unique illiquid asset $a_{j,t}$ with nominal return $r_{j,t}$. Since capital is sector-specific, we assume that households from sector j can only possess capital from sector j. Agent of type j maximize their inter-temporal discounted utility by choosing their consumption allocation path:

$$\max_{\{c_t^j\}_{t \geq 0}} \mathbb{E}_0 \int_0^{+\infty} e^{-\rho t} u(c_t^j) \, \mathrm{d}t \tag{8}$$

where c_t^j is the consumption basket of the household j in real terms and $\rho \in (0, 1)$ is the discount factor. The utility function u is CRRA[4] and the consumption basket is the same CES aggregator across types such that:

$$c_t^j = \left(\sum_{i=1}^{N} \omega_i^{1/\sigma} c_{it}^{j(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{9}$$

where $c_{i,t}^j$ is the demand from consumer j for good i, ω_j captures the relative share of the j-th industry's final goods in the preferences of the consumer and σ is the final demand elasticity of substitution across goods. Notice that we assume that all types of households have the same preferences. Thus, the households exhibit earnings heterogeneity but

[4] u is such that: $u(c) = \frac{c^{1-s}}{1-s}$, $\forall c$

preferences homogeneity. The corresponding ideal price index P_t is such that:

$$P_t = \left(\sum_{j=1}^{N} \omega_j P_{j,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} . \tag{10}$$

Defining the real prices $r_{j,t}^r := \frac{r_{j,t}}{P_t}$ and $w_{j,t}^r := \frac{w_{j,t}}{P_t}$, the budget constraint of a household working in sector j writes down in real terms:

$$\dot{a}_{j,t} = r_{j,t}^r a_{j,t} + w_{j,t}^r z_{j,t} - c_t^j. \tag{11}$$

where $a_{j,t}$ is the sector j households' choice of real asset and $z_{j,t}$ is the vector of labour productivity. Slightly abusing notations, we use the simpler a and z when the indexation on time and sectors is obvious. Notice that we do not explicitly multiply the labor income by the number of hours worked (which is fixed and equal to $\tilde{L}_j$) but internalize this value in the labor productivity. For simplicity, we assume that idiosyncratic labor productivity has only two states $z_{j,t} \in \{z_j^1, z_j^2\}$ for each sector j and that these states follow a simple Poisson process with intensities $\lambda(ii')$, where $i = 1, 2$. The borrowing constraint for each individual ensures that:

$$a_{j,t} \geq \underline{a_j} \tag{12}$$

where $-\infty < \underline{a_j} \leq 0$ is the illiquid asset borrowing constraint. The density functions $g_j(a, z, t)$ describe the distribution of wealth and productivity for the households at each period. The maximization program of the household involving a consumption-saving path and the evolution of the joint distributions $(g_j)_j$ yields two set of differential equations, respectively a Hamilton-Jacobi-Bellman equation (HJB) and a Kolmogorov Forward (KF), one for each sector j:

$$\rho v_j(a, z, t) = \max_{\{c_t^j\}} u(c_t^j) + \partial_a v_j(a, z, t)(r_{j,t}^r a_{j,t} + w_{j,t}^r z_{j,t} - c_t^j) + \sum_{i'} \lambda(ii') v_j(a, z, t) + \partial_t v_j(a, z, t) \tag{13}$$

$$\partial_t g_j(a, z, t) = -\partial_a [g_j(a, z, t) s_j(a, z, t)] + \sum_{i'} \lambda(ii') v_j(a, z, t) \tag{14}$$

where $s_j(a, z, t) := r_{j,t}^r a_{j,t} + w_{j,t}^r z_{j,t} - c_t^j$ is the saving policy function.[5]

[5] The household j first order condition reads as: $c^j(a, z, t) = u^{-1}(\partial_a v_j(a, z, t))$.

2.3 Market clearings

There are $3N$ equilibrium conditions. Firstly, the capital market must clear. Since capital is sector-specific and possessed by households, each sector j must verify at all period t:

$$\int_{\underline{a}}^{+\infty} \int_{z} g_j(a,z,t) a_{j,t} \, \mathrm{d}a \mathrm{d}z = K_{j,t}. \tag{15}$$

The market condition for each good ensures that all produced goods are either consumed, used as intermediate inputs or invested. Thus, for each sector j, the following condition holds:

$$Y_{j,t} = \sum_i M_{ij,t} + I_{j,t} + C_{j,t} \tag{16}$$

where $C_{j,t} = \sum_{i=1}^{N} \int \int g_j(a,z,t) c^i_{j,t} \, \mathrm{d}z \, \mathrm{d}a$ is the total final consumption for good j from all households and $I_{j,t}$ is investment in sector j at time t.

Finally, the distributions of households ensure that labor supply is equal to the cumulated density in each sector:

$$\int_{\underline{a}}^{+\infty} \int_{z} g_j(a,z,t) \, \mathrm{d}z \, \mathrm{d}a = \tilde{L}_j, \ \forall j, t \tag{17}$$

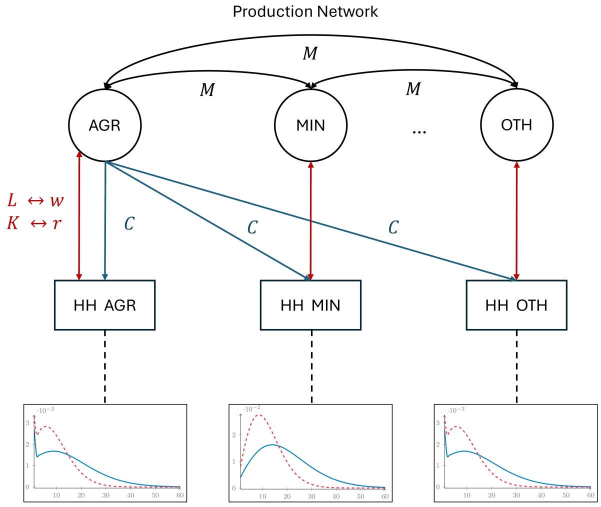
The following scheme gives a bird's eye view of the model and the main linkages between agents and sectors:

2.4 Equilibria definition

We define a stationary competitive equilibrium as follows:

Definition 1 (Stationary Equilibrium) *A stationary recursive competitive equilibrium is defined as:*

1. Value and policy functions for all types of sectoral household j: $v_j(a,z)$, $c_j(a,z)$ and $s_j(a,z)$
2. Sectoral factor and input demands $(K_j)_j$, $(L_j)_j$ and $(M_{ij})_{i,j}$
3. Distributions of household wealth for all sectors j: $g_j(a,z)$
4. Sectoral prices for each good, sectoral interest rates for each type of capital and sectoral wages: $(P_j)_j$, $(r_j)_j$ and $(w_j)_j$



Notes: In this scheme, the production of the various sectors is used as inputs for others (the M), as consumption for all households (the C) and for investment. Households are categorized by the sector they are working for, supplying labor L in exchange of wage w. They also possess the capital K which is rented to firms at rate r. Each type of household is associated with a double distribution, one for each productivity (z_1,z_2).

Figure 2: Summary scheme of the model

such that:

1. Given a set of prices $(P_j)_j$, $(r_j)_j$ *and* $(w_j)_j$, *each type of household* j *value function* $v_j(a,z)$ *solves the household problem, namely the stationary HJB equation :*

$$\rho v_j(a,z) = \max_{\{c^j\}} u(c^j) + \partial_a v_j(a,z)(r_j^r a_j + w_j^r z_j - c^j)$$

2. Given a set of prices $(P_j)_j$, $(r_j)_j$ *and* $(w_j)_j$, *the factor and input demands* $(K_j)_j$, $(L_j)_j$ *and* $(M_{ij})_{ij}$ *solve the sector's maximization problem, thus the first order conditions detailed above*

3. For all sector j, *given the saving policy function* $s_j(a,z)$, *the distribution* $g_j(a,z)$ *satisfies the stationary KF equation :*

$$0 = -\partial_a[g_j(a,z)s_j(a,z)]$$

4. Given the distributions $(g_j(a,z))_j$, *all the the markets for capital, goods and labor clear (namely Equation 15, Equation 16 and Equation 17).*

Accordingly, we define the time-dependent analog of equilibrium as follows:

Definition 2 (Time-dependent equilibrium) *The time-dependent analog of the re-*

cursive stationary equilibrium is defined as:

1. Value and policy functions for all types of sectoral household j: $v_j(a,z,t)$, $c_j(a,z,t)$ and $s_j(a,z,t)$

2. Sectoral factor and input demands: $(K_{j,t})_j$, $(L_{j,t})_j$ and $(M_{ij,t})_{i,j}$

3. Distribution of household wealth for all sectors j: $g_j(a,z,t)$

4. Sectoral prices for each good, sectoral interest rates for each type of capital and sectoral wages: $(P_{j,t})_j$, $(r_{j,t})_j$ and $(w_{j,t})_j$

such that:

1. Given a set of prices $(P_{j,t})_j$, $(r_{j,t})_j$, $(w_{j,t})_j$ and a terminal condition for the value function $v_j^{\infty}(a,z)$, each type of household j value function $v_j(a,z,t)$ solves the household problem, namely the dynamic HJB equation given by Equation 13 with the terminal condition $lim_{T\hookrightarrow\infty}v(a,z,T) = v_j^{\infty}(a,z)$ at each period t

2. Given a set of prices $(P_{j,t})_j$, $(r_{j,t})_j$, $(w_{j,t})_j$, the factor and input demands $(K_{j,t})_j$, $(L_{j,t})_j$ and $(M_{ij,t})_{ij}$ solve the sector's maximization problem at each period t

3. For all sector j, given the saving policy function $s_j(a,z,t)$ and the initial distribution $g_j^0(a,z) = g_j(a,z,0)$, the distribution $g_j(a,z,t)$ satisfies the dynamic KF given by Equation 14 at each period t

4. Given the distributions $(g_j(a,z,t))_j$, all the the markets for capital, goods and labor clear at each period t (namely Equation 15, Equation 16 and Equation 17).

3 Solution method

This section provides the solution framework for solving our model. We add details about adapting the heterogeneous agents algorithm to our disaggregated production environment.

3.1 Steady-state and transition dynamics

The algorithmic procedure for solving the HACT-IO model can be divided in two phases: finding the stationary equilibrium and computing transition dynamics. The idea is essentially to replicate N times the projection method from Achdou et al. [2022] using aggregate quantities and prices as linkages between the different HJB and KF blocks of the algorithm. The next subsection discusses the specific features of solving a HACT-IO model with respect to a standard heterogeneous-agent model.[6] We discretize time in addition to wealth and income. For each sector, we construct a linearly-spaced asset grid with 110 points. The first step to solving the model is to find its stationary equilibrium. We solve for the steady state by following this 6-steps procedure:

1. Guess the sectoral capital stocks $(K_j)_j$ and the sectoral prices $(P_j)_j$.

2. Compute all sectoral quantities and prices, namely $(Y_j)_j$, $(M_{ij})_{i,j}$, $(r_j^r)_j$ and $(w_j^r)_j$ using the guesses. Computations are provided in Appendix B.

3. Solve the households' dynamic program (HJB equation) using the finite difference method from Achdou et al. [2022]. This step yields the value functions $(v_j(a,z))_j$, the consumption functions $(c^j(a,z))_j$ and the savings functions $(s_j(a,z))_j$.

4. Solve for the joint distributions of households KF equation using the finite difference method from Achdou et al. [2022]. This step yields the density functions $(g_j(a,z))_j$.

5. Compute the residuals of the capital market clearing and goods market clearing (namely Equation 15 and Equation 16).

6. Update the guesses using any optimization routine or using a bisection method until residuals are low enough. Additional details on this step are given in the next paragraph.

The procedure to compute the transition dynamics is essentially identical as the one for the steady state, except that one needs to guess the entire capital and prices paths, $(K_{j,t})_{j,t}$ and $(P_{j,t})_{j,t}$. HJB and KF equations are solved at each period t and the entire path of residuals must be cleared.

3.2 Accounting for the Input-Output architecture

At first glance, the solution method is very similar to the method used for standard heterogeneous-agents model in continuous time. Nevertheless, the high dimensionality associated with the disaggregation of the production sharply increases the complexity of the computations. To that extent, we give in this subsection additional details on the potential hindrances faced when solving an HACT-IO model and we provide solutions to overcome these obstacles.

Sensitivity and choice of guesses. Continuous-time heterogeneous-agent models can be very sensitive to the choice of the first guesses. As our procedure implies to guess $2N$ variables (capital and prices), finding a good region for the initial guesses

[6] Such as an Aiyagari [1994] model.

becomes tricky. We suggest to adopt the following incremental procedure. We start with a perfectly symmetric set-up. In other words, we find the equilibrium capital and price that solves for a calibration where all sectoral parameters are equal across sectors and the input-output matrix is chosen to be symmetric. As a result, one only needs to guess 2 variables, as capital and prices must be equal across sectors. Sectoral parameters are denoted $\mathbf{\Theta}_0$ and the IO matrix is denoted Γ_0, such that the initial equilibrium is a couple $(K^*(\mathbf{\Theta}_0, \Gamma_0), P^*(\mathbf{\Theta}_0, \Gamma_0))$. Conversely, denote $\mathbf{\Theta}$ and Γ the sectoral parameters and the IO matrix of the aimed calibration. The idea is to successively solve the model for values of the parameters on the segment $[(\mathbf{\Theta}_0, \Gamma_0), (\mathbf{\Theta}, \Gamma)]$, with incremental changes at iteration. As a result, it is possible to use the last equilbrium found as a guess for the next incremental change in parameters. Formally, divide the segment $[(\mathbf{\Theta}_0, \Gamma_0), (\mathbf{\Theta}, \Gamma)]$ in n linearly spaced values, such that for $i = 1, ...n$, parameters are defined by:

$$\mathbf{\Theta}_i = (1 - \frac{i}{n})\mathbf{\Theta}_0 + \frac{i}{n}\mathbf{\Theta}, \quad \Gamma_i = (1 - \frac{i}{n})\Gamma_0 + \frac{i}{n}\Gamma$$

When the equilibrium is found for iteration i, the equilibrium values for capital and prices can be used as guesses for the iteration $i + 1$, because the variation in parameters is small enough to preserve stability.[7]

Sectoral grids. Projection methods such as in Achdou et al. [2022] require to discretize wealth. In our case, we use a linearly-spaced asset grid. However, using the same grid for every sector impairs the stability of the model. For example, if sector A uses 10 times more capital than sector B, the distribution of wealth in sector A starkly differs from that in sector B. In particular, households in sector A tend to be richer than households in sector B (before dividing by the size of the population in each sector). Thus, to ensure the stability of the computation of the wealth distributions in the KF block, we use sector-specific grids that are proportional to the capital use in each sector. To be specific, we set the upper bound of wealth grids to be five times larger than the sectoral capital: $\overline{a_j} = 5K_j \ \forall j$.

3.3 Calibration

We calibrate our model on the US economy using the 2-digit level from the NAICS classification excluding the government sector. This specification yields a production network with 14 sectors. The data is taken from the BEA Input-output tables. The list

[7]Note that choosing a high value for n does not necessarily increase a lot the computation time: while there are more iterations, each of them is faster to solve than for larger increments of parameters.

of the 14 sectors can be found in Appendix.

The calibration of the Input-Output parameters $(\gamma_{ij})_{i,j}$ is standard (Foerster et al., 2011, Atalay, 2017...) and exploits the Commodity-by-industry defined from the BEA's Input-Output tables. The tables between 2001 and 2020 are used to calculate the average factor shares over these 20 observations to calibrate our production network. The sectoral elasticities of substitution $(\sigma_j)_j$, the labor factor intensity parameters $(\beta_j)_j$ and the final demand shares $(\omega_j)_j$ are directly taken from Poirier and Vermandel [2024]. The capital intensity parameters are taken from Schaab and Tan [2024]. At the sectoral level, we divide the intensity parameters by their sum to ensure constant returns to scale.

The remaining calibration of macroeconomic parameters is standard. We set the discount rate to $\rho = 0.05$ and the capital depreciation rate to $\delta = 0.285$. The elasticity of substitution for final consumption is set to $\sigma = 0.6$ as suggested by the findings of Atalay [2017] or Poirier and Vermandel [2024]. The coefficient or risk aversion of the CRRA utility function is set at $\mu = 2$ and we exclude borrowing possibilities such that the borrowing limit is the same for all sectors: $\underline{a}_j = \underline{a} = 0 \ \ \forall j$

Parameter	Value	Description	Source/Target
ρ	0.05	Discount factor	Achdou et al. [2022]
δ	0.0285	Capital depreciation rate	Achdou et al. [2022]
σ	0.6	Elasticity of final demand substitution	Atalay [2017]
μ	2	CRRA Risk aversion	Standard
$\underline{a}_j = \underline{a}$	0	Borrowing limit	Standard
$(\alpha_j)_j$	See Appendix A	Capital intensities	Schaab and Tan [2024]
$(\beta_j)_j$	See Appendix A	Labor intensities	Poirier and Vermandel [2024]
$(\gamma_{ij})_{i,j}$	See Appendix A	Interm. inputs intensities	BEA
$(\sigma_j)_j$	See Appendix A	Sectoral elasticities	Poirier and Vermandel [2024]
$(\omega_j)_j$	See Appendix A	Final demand shares	Poirier and Vermandel [2024]

Table 1: Calibration of model parameters

4 Sectoral shock propagation in HA-IO

4.1 Sectoral classification: Influence vs. Bonacich

In this subsection we discuss some sectoral properties that matter for our understanding of shock transmission. In particular, we classify sectors according to two main measures, Bonacich centrality and sectoral influence. These measures allow us to assess the importance of the different industries in the production network as well as for the households' final consumption. By computing these measures, we detail the reasons why we choose to dissect the propagation of shocks originating from specific sectors.

Leontief inverse. Before defining the two measures mentioned above, we need to define the Leontief inverse of the economy. The Input-Output matrix $\Gamma = (\gamma_{ij})_{i,j}$ expresses the importance of one sector to another as a direct supplier. However, the complex nature of production networks implies that industries also exhibit indirect linkages. For this purpose, we define the Leontief [1942] inverse $L = (l_{ji})_{ji}$, an object that considers all direct and indirect linkages in the economy:

$$L = I_N + \Gamma + \Gamma^2 + ... = \sum_{k=0}^{+\infty} \Gamma^k = (I_N - \Gamma)^{-1} \tag{18}$$

The Perron-Frobenius theorem ensures the existence of such an inverse for the input-output matrix. The element l_{ij} of the Leontief inverse, embeds how important is sector j in supplying sector i through all direct and indirect paths. This matrix is a core determinant of the centrality measures we define thereafter.

Bonacich centrality. While different notions of centrality co-exist in network theory, the Bonacich [1987] centrality (also called "Katz centrality") is particularly well fitted in economic environment. The Bonacich centrality ν_j of a sector j can be expressed in recursive form, or directly using the Leontief inverse matrix:

$$\nu_j = 1 + \sum_i \gamma_{ij}\nu_j = \sum_i l_{ij} \tag{19}$$

The interpretation of centrality is straightforward: a sector is more central in the production network if it is a more important input supplier to other central sectors. Note that the Bonacich centrality is a purely "network-oriented" object and does not tell anything about the importance of a sector for final consumption.

Bonacich centrality matters when considering shock propagation as shocks to more central industries tend to infuse much more in the production network, triggering larger aggregate effects. The left panel of Figure 3 plots the Bonacich centrality of our calibrated economy with 14 sectors in ascending order. We notice a clear gap between the top 3 central sectors (in red) which are the Finance sector, the Business services and the Manufacturing sector. The remaining sectors exhibit relative heterogeneity but this feature suggests that these 3 sectors are particularly crucial in shock propagation.

Sectoral influence. Sectoral influence parameters $(\lambda_j)_j$ are defined in Liu [2019] as:

$$\lambda_j = \sum_i l_{ij}\omega_i \tag{20}$$

In a perfectly competitive equilibrium such as in our set-up, Liu [2019] notes that sectoral influence can be viewed as "expenditure-based centrality measures of equilibrium sectoral size". As opposed to Bonacich centralities, sectoral influence express how important is a sector as a direct and indirect supplier of the economy *in general*, including sectors *and* households. While Bonacich centrality only informs on characteristics inside the network, sectoral influence also take into account the final demand for sectoral goods and services originating from consumers. If we assume that the househols' preferences are Cobb-Douglas, a standard result[8] is that the sectoral influence of sector j is equal to its Domar [1961] weight, *i.e.* the ratio of this sector's sales on GDP:

$$\lambda_j = \frac{P_j Y_j}{GDP} \tag{21}$$

Hulten [1978]'s foundational theorem states that, to the first-order, the impact of a supply shock in a specific sector on GDP is equal to its Domar weight. Thus, Domar weights are critical when considering microeconomic shock propagation. In our model, the specification is significantly different (*e.g*: CES preferences, heterogeneous agents) and we use projection method as opposed to perturbation, such that we compute the responses to shocks without making any approximation. However, sectoral influences still play a similar role in shock transmission than Domar weights, and constitute an important measure to assess shock transmission. The right panel of Figure 3 displays the sectoral influences of our calibrated economy with 14 sectors in ascending order. The same conclusions as for Bonacich centrality apply, with a clear gap between the top 3 sectors and the rest. We notice more heterogeneity inside the top 3 (with a reversal between Finance and Business services) and large variations of rankings between sectors outside the top 3.

[8] See, for instance Carvalho and Tahbaz-Salehi [2019] for a concise proof.

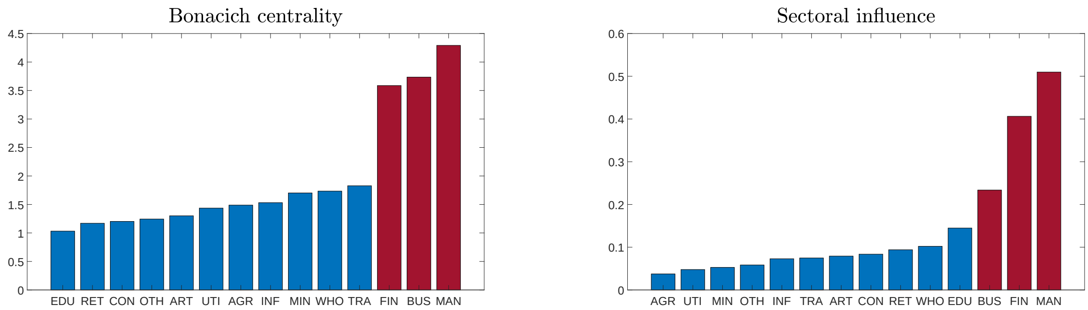


Figure 3: Barplot of Bonacich centrality measures and sectoral influences.[9]

The comparison between the measures encourages the distinction of shock analysis between central sector in the sense of Bonacich and key sectors in the sense of influence. In what follows, we compare the dynamics of our economy following a supply shock in three different types of sectors. First, we consider a shock in the Manufacturing sector (MAN), which is crucial as suggested by both measures. Second, we consider a shock in the Educational services, health care, and social assistance sector (EDU), as this sector is the least central in the sense of Bonacich, but is highly influential on final consumption as suggested by its sectoral influence. Finally, we consider a shock in the Agricultural sector (AGR), which shows to be fairly central in the production network but exhibits the lowest sectoral influence.

Sector	Bonacich centrality	Sectoral Influence
MAN	++	++
EDU	-	+
AGR	+	-

Table 2: Dichotomy of shocks considered

4.2 Effects on prices

We first simulate the effects of sectoral shocks on prices. Figure 4 displays the Impulse Response Functions (IRFs) of real interest rates, real prices and real wages following a 1% negative supply shock originating from one of the three sectors aforementioned. Before considering the heterogeneous effects of sectoral shocks, we discuss some general properties of propagation on prices.

Following a negative TFP shock in a specific industry, the real price of this industry's goods systematically increases, a standard result of the business cycles literature. Conversely, the real prices of other sectors most often decline as a consequence to the increase

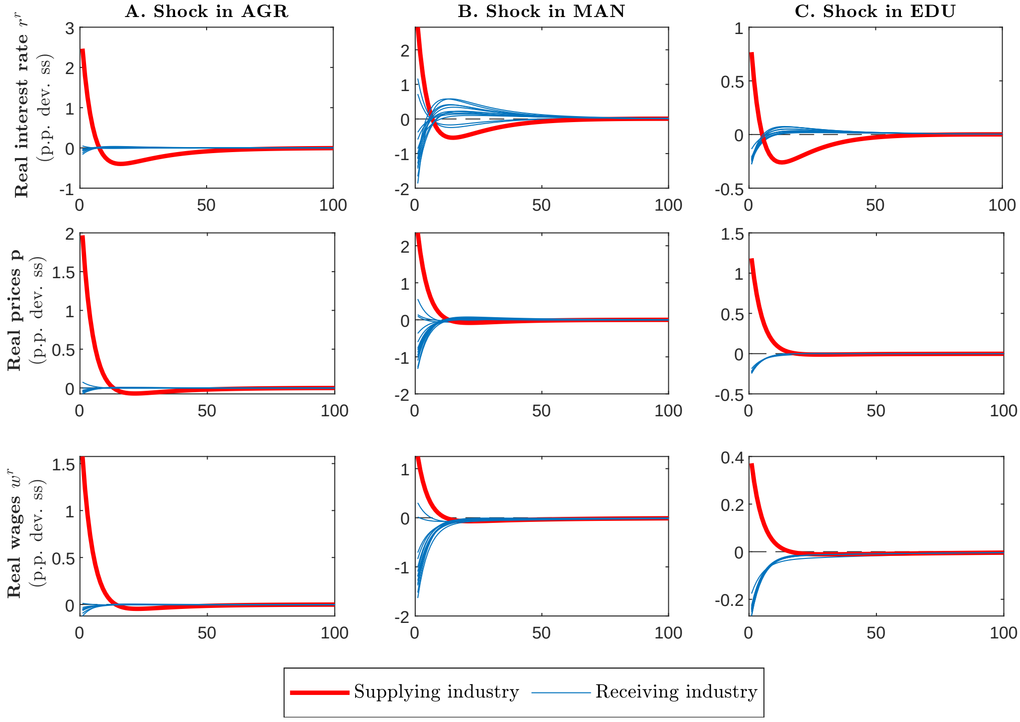


Figure 4: Irfs of real prices following three different types of sectoral shocks

in the aggregate price index.[10] The shock on the supplying industry's real price is always more intense than for the receiving sectors. Negative TFP shocks also generate increases in the real interest rates and real wages of the supplying industry, while triggering opposite dynamics in receiving sectors. The mechanism behind the temporary increase in real wages and real interest rates in the supplying sector is what we call a *price effect*. Suppose that a shock hits sector j, Equation 6 ensures that the real wages in sector j writes down:

$$w^r_{j,t} = \beta_j \frac{p_{j,t} Y_{j,t}}{L_{j,t}}$$

with constant labour supply $L_{j,t} = \tilde{L}_j$. The negative shock in sector j increases the real price of good j and decreases the output due to depressed demand (see next subsection for the effects on quantities). However, the increase in the price of sector j is relatively larger than the decrease in output, such that real wages, which rewards labor at his marginal productivity, increase. This is what we define here as a *price effect*, an

[10] While all sectors' nominal prices increase, the price index increases more, creating a decline in prices in real terms.

increase in sales due to inflation despite depressed demand. This effect also applies to capital.

We next turn to discuss the idiosyncratic properties of shock propagation. In particular, we have chosen to simulate shocks originating from three different types of sectors to assess the characteristics that matter for shock transmission. Unsurprisingly, the Manufacturing triggers much larger co-movements with other industries than other sectors, as it is the most central according to both centrality measures. Figure 4 shows that a 1% negative supply shock in the Manufacturing sector generates decreases in other industries prices up to 2% for real interest rates, 1.3% for real prices and 1.6% for real wages.

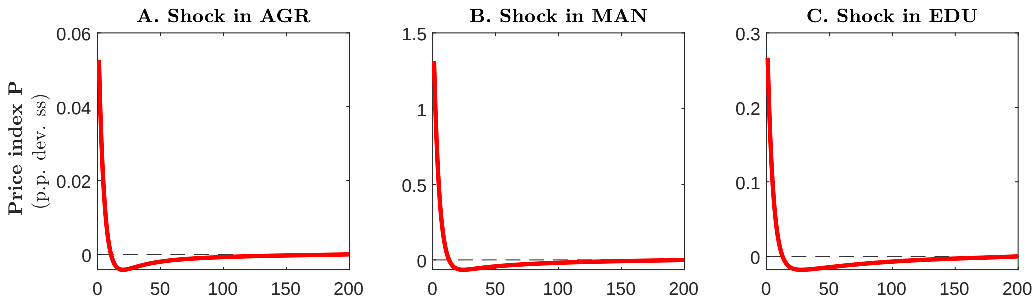


Figure 5: Irfs of price index following three different types of sectoral shocks

The comparison between the first column and the third column of Figure 4 is insightful. The IRFs highlight that a shock in the Educational, health and social services propagates much more through the network than a shock in the Agricultural sector, as it triggers much larger responses from other industries' prices. Yet, the EDU sector is by far the least central in the production network as suggested by the Bonacich centrality measure. The intuition behind this observation is the crucial importance of sector EDU in final demand from the households. The budget share allocated to this sector by households is the third largest. As a consequence, shocks in the EDU sector generate large variations in real prices through backward-demand linkages that can be traced back to the effects on the aggregate price index. Figure 5 shows how the three different shocks yield three different orders of magnitude of price index variation. In particular, a shock in EDU generates variations in the price index almost five times larger than a shock in AGR. These results suggest that sectoral influences matter more than Bonacich centrality in shock propagation on prices.

4.3 Effects on quantities

We next turn to discussing the impacts of sectoral shocks on quantities. Figure 6 displays the IRFs of sectoral capital, sectoral total output and sectoral consumption (*i.e.* output used for consumption from all households), following a 1% negative supply shock from our three key sectors.

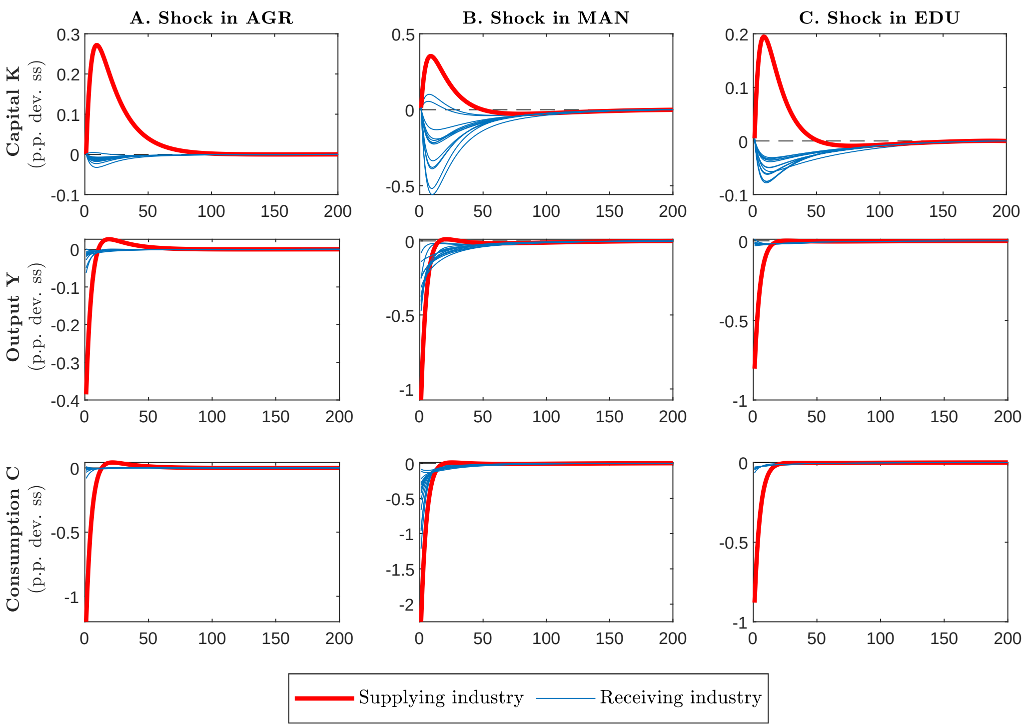


Figure 6: Irfs of quantities following three different types of sectoral shocks

The response of capital to sectoral shocks shows to be opposite if considering the impact on the supplying industry or the receiving industry. The top panels of Figure 6 suggest that a negative supply shock originating from an industry leads to an accumulation of capital in the same industry. Conversely, other sectors' capital tend to diminish before going back to the steady state. Furthermore, the sectoral outputs and sectoral consumptions exhibit shock responses that are fairly identical. A negative TFP shock tends to decrease output and consumption for all sectors, with a higher impact on the supplying sector which suffers the most from the shock.

Figure 6 corroborates the findings from the prices analysis on centrality measures. We find that shocks originating from the Manufacturing industry propagate the most in the network and produce the largest disturbances in quantities. Most importantly, negative supply shocks from Educational services, health care, and social assistance trigger larger decreases in output and consumption than shocks from the Agricultural sector. Figure 7 plots the responses of real aggregate consumption (*i.e:* real GDP) in different cases:

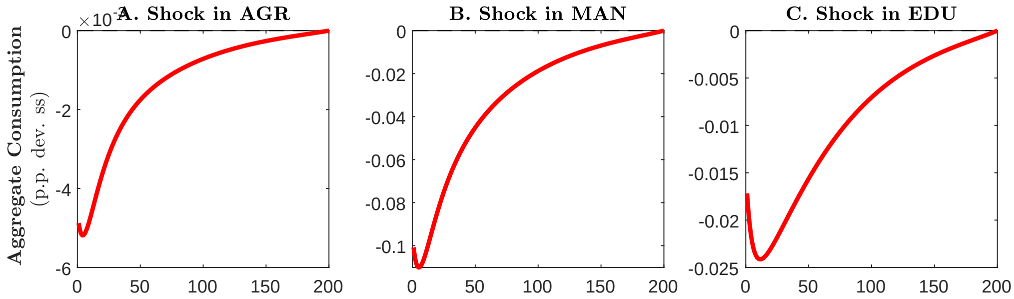


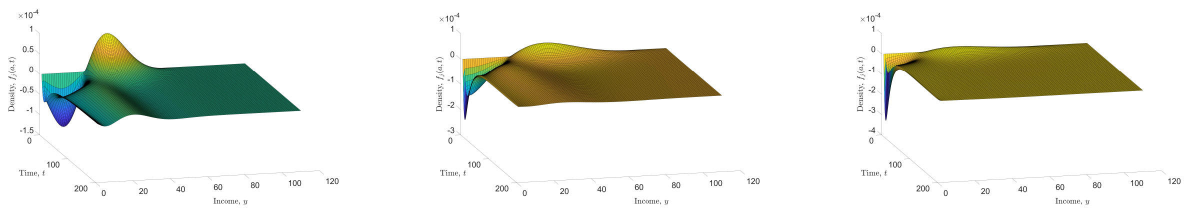
Figure 7: Irfs of aggregate consumption following three different types of sectoral shocks

Ultimately, a crucial output of our analysis is to distinguish the aggregate impacts of different idiosyncratic shocks. Figure 7 shows that sectoral influences are a more accurate centrality measure to capture the macroeconomic consequences of sectoral shocks than the Bonacich centrality. We observe the same pattern as for the aggregate prices, with three distinct orders of magnitude for the impulse responses, the Manufacturing sector being the most decisive and the Agricultural sector being the least significant.

5 The distributional effects of sectoral shocks

5.1 Intra-sectoral income redistribution

A natural question at this stage is how idiosyncratic shocks translate into distributional effects for different types of households and what are the consequences in terms of inequalities. We first focus on income inequalities. We compute on Figure 8 the IRFs of the income distributions of the supplying industry for a 1% negative supply shock originating from our three sectors of interest.



(a) Shock in AGR (b) Shock in MAN (c) Shock in EDU

Figure 8: Evolution of income distribution over time for supplying sectors

While the magnitude of the responses differ from a sector to another, we observe that the income redistribution patterns are very similar across industries. On the short run, the income distributions of supplying sectors primarily hit by the shock shift up, with a stark decline in the proportion of households located close to the borrowing constraint. Conversely, the concentration of households in the middle of the distribution substantially increases. This shift in the distribution can be attributed to the enrichment of the sector with higher real wages, higher real interest rates and higher capital (see Figure 4). These dynamics result in higher returns of labor and capital, overall enriching all the distribution of household in the supplying sector. On the long run, these redistribution effects are reversed and offset. For instance in the Manufacturing sector, we observe an increase in the concentration of households in the first decile 60 periods after a shock in the same sector. This impoverishment is due to a readjustment of real prices, with real interest rates and real wages lower than their steady sate counterpart.

To assess for income inequality changes within sectors, we plot in Appendix C the IRFs of sectoral Income Gini index following 1% negative supply shocks.[11] Our results suggest that, on the short run, shocks originating from a sector increase income inequalities in that sector as it is shown by an increase in the Gini index. On the medium and long run, the trends reverse and the Gini index stays below the steady state level. [12] In other sectors, income inequalities increase on any horizon considered. These distributional costs are attributed to differences in the returns on capital and labor as explained below with a simplified example.

To understand why income inequalities increase in the supplying sector, take two households working for the same sector and with the same productivity normalized to 1. Suppose now that the first household has wealth $a = \underline{a}$ and earnings $\underline{y} = w + r\underline{a}$, and suppose that the second household has wealth $\overline{a} > \underline{a}$ such that its income writes down $\overline{y} = w + r\overline{a}$. Following a shock, the variation of income for the first household is $\Delta\underline{y} = \Delta w + \underline{a}\Delta r$ while the variation of income from the latter household is $\Delta\overline{y} = \Delta w + \overline{a}\Delta r$. Thus, the relative variation in income of the rich household is larger if:

$$\frac{\Delta\underline{y}}{\underline{y}} \leq \frac{\Delta\overline{y}}{\overline{y}} \Longleftrightarrow \frac{\Delta w}{w} \leq \frac{\Delta r}{r} \tag{22}$$

[11]Gini indexes are widely used for inequality measures. A Gini index equal to 0 corresponds to perfect equality in the economy while a Gini index equal to 1 corresponds to absolute inequality, with a single individual owning all wealth or earning all income.

[12]This switch is concordant with the "undershoot" of other variables when they return to the steady states (see the IRFs of prices in Figure 4).

This simplified example shows that the relative income increase of richer households is larger to that of poorer households if the increase of returns on capital exceed the increase in wage inflation. In section 4, we observe that this inequality holds for all shocks in the supplying industry (at time $t = 0$), which rationalizes the increase in income inequalities in the supplying sector.

5.2 Intra-sectoral wealth redistribution

Regarding wealth distribution, Figure 14 in Appendix C displays the IRFs of Wealth Gini indexes following the negative supply shocks. The patterns of the wealth Gini responses are close to that of income Ginis for the receiving industries, with increasing wealth inequalities. As for the supplying industry, the dynamics are very similar, but the wealth inequalities immediately reduce after the shock (while they first increase for income). This finding suggests that negative supply shocks generate positive redistribution effects for the industry experiencing the shock, and substantial distributional costs for downstream sectors. As detailed below, this phenomena is attributable to the income dynamics, with a sharp increase in real wages in the supplying sector and the decline in real wages in receiving industries.

In a similar fashion as for income inequalities, we inspect the mechanisms behind wealth redistribution through the lens of a simplified toy example. Consider the same two households as in the previous subsection. Suppose a variation in wealth following a shock. The poorer household's wealth variation is $\Delta\underline{a} = \Delta w + \underline{a}\Delta r - \Delta\underline{c}$ while the richer household's wealth variation is $\Delta\overline{a} = \Delta w + \overline{a}\Delta r - \Delta\overline{c}$. Thus, the relative variation in wealth of the rich household is lower if:

$$\frac{\Delta\underline{a}}{\underline{a}} \geq \frac{\Delta\overline{a}}{\overline{a}} \Longleftrightarrow \frac{\Delta w - \Delta\underline{c}}{\underline{a}} \geq \frac{\Delta w - \Delta\overline{c}}{\overline{a}} \tag{23}$$

This condition is verified as soon as richer households consume more than poorer households (in level, not relatively). Hence, the increase in income following a negative supply shock in the supplying sector leads to a relatively larger increase in poorer households' wealth. This in turn, reduces wealth inequalities by increasing the density of middle-wealth households.[13]

[13]This mechanism becomes more intuitive when thinking of a household located at the borrowing constraint. His wealth is zero ($a = 0$), such that an increase in his income allows him to partly save because his marginal propensity to consume is slightly less than 1. The relative increase in wealth is infinite.

What sectors matter the most for wealth redistribution ? The Gini responses from Figure 14 clearly corroborate the results from Section 4. Similar as for prices or quantities, disturbances in the Manufacturing sector have the larger implications on inequalities. A 1% shock in this sector triggers an increase of the wealth Gini index up to 0.4% for other sectors, implying sizeable distributional costs for other industries. In addition, the distributional impacts are considerably larger for shocks emanating from the EDU sector than from the Agricultural industry. They are larger for both supplying and receiving industries, as can be observed from the maximal magnitude of the responses. This observation suggests that sectoral influence is also key in capturing the importance of a sector for redistribution.

5.3 Consumption dynamics by deciles

Our model is amenable to disentangle the consumption responses of different deciles for each sector's wealth distribution. Figure 9 plots the real consumption IRFs of four different deciles depending on wealth (Top 10%/Bottom 10%) and productivity (low productivity z_1/high productivity z_2), in the sector experiencing the original shock.

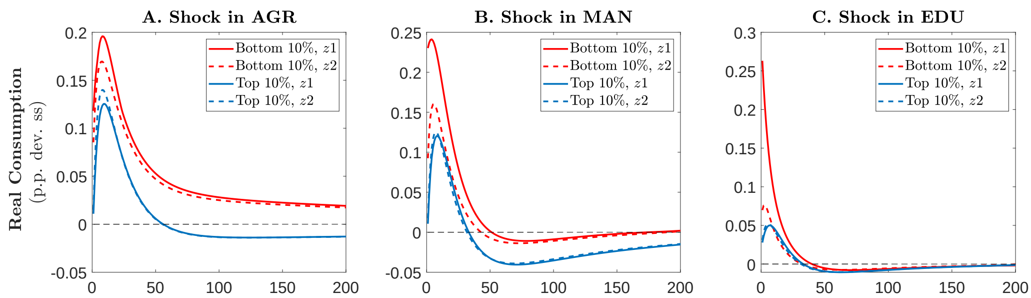


Figure 9: IRFs of real consumption in supplying sectors by top and bottom deciles and productivity.

Two observations are worth discussing. First, we find that the deciles' consumption patterns are very akin across supplying sectors, with an initial spike in consumption due to the increase in income. In particular, the order of magnitude of the different consumption responses are similar. This suggests that consumption in the shocked sector behaves in an analogous fashion, irrespectively of the industry considered. Second, consistent with the literature on marginal propensities to consume (MPCs), we find that consumption is negatively correlated with the level of wealth.[14] The consumption of households in the first wealth decile (Bottom 10%) rises significantly more than the consumption of rich households (Top 10%). Intuitively, the former households are strongly budget constrained, such that a sudden increase in their income allows them to consume more than their current state of wealth.

6 Fiscal policy

In this section, we investigate the sectoral and distributional implications of tax changes by introducing a government which acts as the fiscal authority.

6.1 A model with public expenditures

The model is essentially similar as that described in Section 2. We extend the latter by considering a government, which has two fiscal instruments at its disposal: a labor tax τ_L and a capital tax τ_K. The labor income tax τ_L is based on real wages while the capital tax τ_K applies to the real income generated by capital renting to firms. Both taxes are distorting and homogeneous across sectors, such that the real budget constraint (for any consumer in sector j at time t) is now:

$$\dot{a}_{j,t} = (1 - \tau_K) r^r_{j,t} a_{j,t} + (1 - \tau_L) w^r_{j,t} z_{j,t} - c^j_t. \tag{24}$$

Using the market clearing for capital, the tax revenues T_t levied by the government are computed as:

$$T_t = \tau_K \sum_j r^r_{j,t} K_{j,t} + \tau_L \sum_j w^r_{j,t} L_{j,t} \tag{25}$$

The government spends the tax revenues in public expenditures $(G_{j,t})_j$ which are derived from the maximization of the state's preferences. The government is supposed to maximize a Cobb-Douglas public expenditure function G_t, such that:

$$G_t = \prod_j G^{\psi_j}_{j,t} \tag{26}$$

[14]Note that Figure 9 does not plot MPCs *stricto sensu*. However, MPCs and consumption IRFs following unanticipated shocks such as in our model are closely related.

where $(\psi_j)_j$ are the government's spending shares. The maximization yields the sectoral expenditures for good j:

$$G_{j,t} = \psi_j \frac{T_t}{P_{j,t}} \tag{27}$$

Finally, the resource constraint of sector j writes down:

$$Y_{j,t} = \sum_i M_{ij,t} + I_{j,t} + C_{j,t} + G_{j,t} \tag{28}$$

Therefore, the fiscal policy has an ambiguous effect on the sectoral outputs. When taxes increase, the post-tax income of households decrease and consumption is depressed. However, the government's revenues increase and the state purchases more goods from all sectors. What drives the overall impact of such a policy change is the importance of each sector both as a final good for households *and* for the government. Table 3 gives the calibration of fiscal parameters:

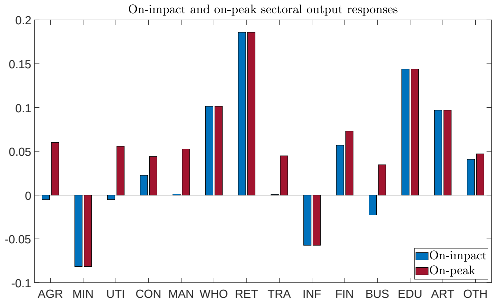
Parameter	Value	Description	Source/Target
τ_K	0.36	Capital tax	Trabandt and Uhlig [2011]
τ_L	0.28	Labor tax	Trabandt and Uhlig [2011]
$(\psi_j)_j$	See Appendix A	Sectoral government spending share	Schaab and Tan [2024]

Table 3: Calibration of fiscal parameters

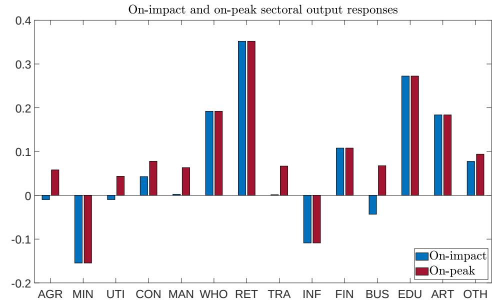
6.2 Sectoral output responses to fiscal shocks

In what follows, we consider two different types of tax shocks. We first compute the transition dynamics in our model when the government decrease the capital tax τ_K by 1 percentage point (from 36% to 35%) at time $t = 0$ before sluggishly readjusting it to 36%. In a second time, we compute the exact same shock for the labor income tax (with a switch from 28% to 27% at time $t = 0$).[15] Figure 10 shows the on-impact and on-peak sectoral output deviations following both shocks:

[15]These shocks can be understood as public spendings cut, since there is no predetermined public budget in the model.



(a) Capital tax shock



(b) Labor tax shock

Figure 10: On-impact and on-peak sectoral output deviations following tax shocks

Public vs Private trade-off. The government's tax cut shifts resources from the state to the households. By decreasing taxes, the fiscal authority levies a lesser proportion of value added but stimulates private consumption by increasing the agents' purchasing power. Thus, the impact on the sectoral outputs is crucially depending on the relative importance of each good in the government and private agents' preferences. To better grasp this trade-off, we simply define the Public-to-Private ratio of a sector j:

$$\chi_j = \frac{\psi_j}{\omega_j} \tag{29}$$

When the ratio χ_j is high, goods produced by sector j are more important for the government than for final consumption. In the case where $\chi_j = 1$, private and public agents equally value good j in their preferences.

Intuitively, the outputs from sectors with low Public-to-Private ratio tend to increase the most following tax cuts since consumption is shifting from the public sector to the households. This is the case for sectors such as Retail trade, Wholesale Trade and Educational services, health care, and social assistance, the three sectors exhibiting the lowest χ_j.[16] Conversely, the Mining sector and the Information sector represent a significant share of public expenditures but a very low share of private spendings. As a consequence, the eviction of consumption from public to private strongly depresses the final demand for these goods. A scatter-plot showing on-peak output multipliers and Public-to-Private ratios is available in Appendix D, which exhibits a clear negative relationship between these variables.

[16]Note that Retail and Wholesale sectors have ratios equal to 0 since their goods are not purchased by the government.

Network effects. Public-to-Private ratios is not the only characteristics that drive the sectoral activities after tax cuts. While the Transportation sector, the Business services or the Utilities services have Public-to-Private ratios greater than one, the on-peak impact of the tax cut is positive for all of them. The network's structure is key in propagating the tax shock through input-output linkages. The most central industries in the network benefit from the increase in final demand for other sectors, which triggers higher demand for their goods as input needs. This channel of input demand plays a major role for sectors such as Business services or Manufacturing.

Capital or labor tax cut. Two remarks are worth mentioning when comparing capital tax and labor tax cuts. First, the patterns exhibited by the different tax shocks are very similar. This concordance follows from the akin distorting nature of the taxes, which enter the model in a quasi-identical way. The program of the firms are unchanged and public expenditures are decided irrespectively of the fiscal revenues breakdown. As for the households, consumption-saving decisions only depend on the amount of post-tax income. All in all, the dynamics of the economy after tax cuts must be qualitatively similar. Second, we notice that labor tax cuts generate larger fluctuations than capital tax cuts. A first explanation for this observation is the relatively stronger tax cut for labor income.[17] In addition, wages represent a larger proportion of household's income.[18] Hence, a decrease in the labor tax generates a larger increase in the household's budget (in absolute terms) than a similar decrease in the capital tax.

6.3 Fiscal shocks and redistribution heterogeneity

We have argued in the previous subsection that fiscal cuts triggered heterogeneous sectoral output responses. In this subsection, we study the distributional impacts of such tax shocks. Figure 11 plots the temporal evolution of secoral wealth Gini indices following unitary percentage point decrease in capital and labor taxes:

[17]The steady-state labor tax rate is 28%, against 36% for the capital tax. A unitary percentage point decrease in taxes implies a relatively larger tax cut for labor tax than for capital tax.

[18]Estimates of the share of labour compensation in GDP from the FRED database are slightly under 60%. Our model yields a share of 63.5%.

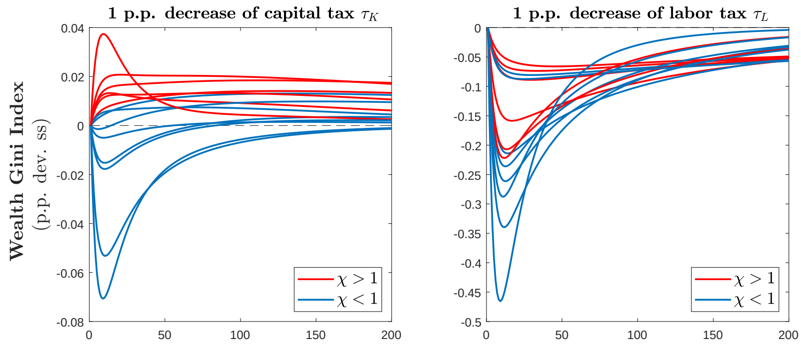


Figure 11: IRFs of sectoral wealth Gini index following tax cuts

Labor tax. Following a cut in the labor income tax, we notice that wealth inequalities decrease in all sectors. The intuition is that the decrease in labor tax generates an income effect, which boosts the households' budget. Agents are able to keep a larger proportion of their labor income, which in turn leads poorer households to save substantially more money and accumulate wealth. This dynamic shifts the distribution to the right in all sectors, even those which work in sectors with high Public-to-private ratios, as the increase in budget (more than) compensates the decline in the sector's activity. However, we notice that the inequalities for households working for sectors with high χ (in red on the figure), tend to diminish to a lesser extent.

Capital tax. Wealth inequality dynamics are much more complex when we consider a capital tax cut. As plotted in Figure 11, the distributional impacts are opposites depending on the sectors considered. A starking feature is the fact that sectors with the highest Public-to-private ratios experience rising inequalities while sectors with low-χ sectors tend to benefit from the tax cut. The intuition is the following. Consider a sector j with a high Public-to-private ratio. The shrinking of public expenditures depresses demand for sector j's good or services, such that real wages and real interests rates drop. However, households are able to keep a larger proportion of their capital income due to the tax cut. The variation of income for a household with wealth a (abstracting from productivity terms) is thus:

$$\Delta y = a \left[\underbrace{(1-\tau_k)\Delta r_j^r}_{-} \underbrace{- r_j^r \Delta \tau_k}_{+} \right] + \underbrace{L_j \Delta w_j^r}_{-} \tag{30}$$

The variation of the return on assets (*i.e.* the term in brackets) is positive, as the tax cut overcomes the decrease in the real interest rate. The variation of labor income is negative.[19]. Thus, there exist a cut-off wealth $\tilde{a}$ such that:

$$\Delta y \geq 0 \iff a \geq \tilde{a} := \frac{-L_j \Delta w_j^r}{(1-\tau_k)\Delta r_j^r - r_j^r \Delta \tau_k} \tag{31}$$

In other words, households with wealth lower than $\tilde{a}$ experience a decrease in their income due to a strong income dependency on labor revenues. This income decline impoverish them and shifts their position to the left of the wealth distribution. Conversely, households with wealth above the threshold move up the distribution as their income increases. In sector j, these forces result in a densification of households close to the borrowing constraint and to the middle of the distribution, which leads to higher inequalities. Figure 16 in Appendix D plots the evolution of the wealth distribution following the capital tax cut in three sectors (MIN, WHO and INF). The distribution in Mining and Information, two sectors with high Public-to-private ratios, clearly corroborate the explanation given above, with an increase in the left bound and the middle of the distribution. For sectors with low χ, real wages increase due to higher demand for the goods or services. It is thus straightforward that inequalities diminish as labor *and* capital income increase, yielding the distribution to shift to the right for all households.

These results shed light on the inherent complexity of designing policies in multi-sectoral environments. While manipulations of the labor tax yield similar distributional outcomes across sectors in our model, variations of the capital tax necessarily deteriorate inequalities in some sectors.

7 Conclusion

This paper takes a step towards modelling heterogeneity in production and final demand by setting out a heterogeneous-agents input-output model in continuous time, which allows us to analyse the distributional impacts of sectoral shocks. The key takeaways from our analysis and findings are as follows.

First, we find that key sectors in shock transmission can be identified using the sectoral influence centrality measure. Shocks originating from industries that exhibit high sectoral influence are more likely to propagate along the production network and create sizeable

[19] Remember that labor supply is inelastic.

fluctuations at the sectoral and aggregate levels. In particular, responses of real GDP to sectoral shocks are proportional to the sectoral influence of the considered industry. Second, we find that negative supply shocks generate positive redistribution effects for the supplying sector, while receiving industries suffer distributional costs. These redistribution effects can be attributed to *income effects*, as real wages and interest rates increase in the supplying sector due to higher sales. The results also corroborate the importance of sectoral influence as a key measure, since industries with high sectoral influence also generate the highest redistributive effects. Third, extending the model by introducing labor and capital income taxes, we find that tax cuts generate very heterogeneous effects on sectoral outputs. We also determine that labor income tax cuts generate a decline in wealth inequalities in all sectors, while capital income tax cuts trigger heterogeneous distributional impacts.

Our model paves the way for exciting research on the linkages between households and sectors, in which we identify natural avenues for improvement. A crucial caveat of our model is the inelasticity of labor supply. Labor reallocation from an industry to another is an inherent feature of sectoral business cycles that is sorely missing in our model. We cannot capture how fluctuations in wages incentivize households to change the industry for which they are working, which is a potentially strong mechanism for shock propagation. We also did not incorporate unemployment in the model. While there are few labor matching models in networks (see Bocquet, 2024 for a recent attempt), the dynamics of unemployment implied by these models are critical when thinking of sectoral cycles. We hope that, in the future, research will address these questions to paint a more accurate portrait of sector-households dynamics.

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Appendix

A Calibration details and additional data

AGR :	Agriculture, forestry, fishing and hunting
MIN :	Mining
UTI :	Utilities
CON :	Construction
MAN :	Manufacturing
WHO :	Wholesale trade
RET :	Retail trade
TRA :	Transportation and warehousing
INF :	Information
FIN :	Finance, insurance, real estate, rental and leasing
BUS :	Professional and business services
EDU :	Educational services, health care, and social assistance
ART :	Arts, entertainment, recreation, accommodation and food services
OTH :	Other services, except government

Table 4: Description of the 14 industries

Sector	K share (α)	L share (β)	Final cons. share (ω)	Prod. elast. (σ)	Gov. share (ψ)	Bona. centr. (ν)	Influences (λ)
AGR	0.287	0.117	0.007	0.441	0.007	1.488	0.037
MIN	0.385	0.150	0.004	0.261	0.029	1.702	0.053
UTI	0.374	0.148	0.018	0.216	0.024	1.436	0.048
CON	0.171	0.332	0.067	0.200	0.062	1.202	0.084
MAN	0.170	0.184	0.233	0.172	0.358	4.292	0.510
WHO	0.257	0.312	0.047	0.339	0	1.734	0.102
RET	0.187	0.371	0.083	0.352	0	1.170	0.094
TRA	0.177	0.303	0.025	0.273	0.033	1.828	0.075
INF	0.330	0.209	0.040	0.373	0.076	1.532	0.073
FIN	0.370	0.149	0.201	0.175	0.129	3.587	0.406
BUS	0.168	0.455	0.037	0.256	0.216	3.736	0.234
EDU	0.114	0.503	0.142	0.348	0.0200	1.032	0.145
ART	0.197	0.342	0.057	0.314	0.019	1.301	0.079
OTH	0.164	0.432	0.041	0.374	0.026	1.243	0.058

Table 5: Sectoral parameters calibration and centrality measures

The input-output parameters (γ_{ji}) are summarized in the following table:

Γ	AGR	MIN	UTI	CON	MAN	WHO	RET	TRA	INF	FIN	BUS	EDU	ART	OTH
AGR	0.32466	0.00074	0.00000	0.00297	0.06790	0.00004	0.00277	0.00017	0.00000	0.00000	0.00124	0.00023	0.01038	0.00068
MIN	0.00814	0.23698	0.27498	0.02142	0.10849	0.00009	0.00009	0.00047	0.00069	0.00001	0.00098	0.00045	0.00229	0.00218
UTI	0.02238	0.03540	0.13476	0.00770	0.02220	0.02381	0.05897	0.03327	0.01023	0.04144	0.01267	0.02054	0.05981	0.01754
CON	0.00676	0.02546	0.03179	0.00037	0.00417	0.00304	0.00661	0.01131	0.00527	0.06284	0.00143	0.00201	0.00624	0.01425
MAN	0.29321	0.24465	0.12009	0.51893	0.54839	0.09153	0.07625	0.22642	0.13049	0.02750	0.10482	0.22988	0.17701	0.23616
WHO	0.12896	0.04857	0.03127	0.09302	0.08970	0.07829	0.03663	0.04631	0.02786	0.01072	0.02089	0.05588	0.03876	0.04209
RET	0.00702	0.00267	0.00946	0.11393	0.00502	0.00122	0.00805	0.02382	0.00107	0.00317	0.00300	0.00138	0.02891	0.03414
TRA	0.03855	0.05520	0.12097	0.03366	0.03983	0.11770	0.13054	0.26370	0.02625	0.01586	0.03662	0.02323	0.02307	0.02057
INF	0.00271	0.00778	0.01590	0.01323	0.00601	0.03671	0.03915	0.01524	0.33528	0.02849	0.06435	0.03157	0.02641	0.04142
FIN	0.14132	0.15132	0.07767	0.06576	0.02593	0.20605	0.27574	0.19065	0.10503	0.53159	0.21911	0.27184	0.21835	0.32304
BUS	0.01811	0.18336	0.15619	0.11499	0.07263	0.38602	0.31761	0.13161	0.28767	0.22568	0.45936	0.24866	0.31107	0.19086
EDU	0.00046	0.00000	0.00106	0.00001	0.00002	0.00258	0.00756	0.00033	0.00060	0.00002	0.00074	0.04134	0.00325	0.00962
ART	0.00287	0.00328	0.01785	0.00159	0.00359	0.01380	0.01312	0.02287	0.05606	0.03175	0.05133	0.04564	0.06568	0.02037
OTH	0.00485	0.00459	0.00801	0.01242	0.00612	0.03912	0.02691	0.03383	0.01351	0.02092	0.02344	0.02734	0.02878	0.04709

Table 6: Calibration values of the Input-Output matrix $\mathbf{\Gamma}$

B Additional computations

B.1 Computation of variables for the algorithm

We give details on the computations of the variables using guesses on the capital and prices from the algorithm described in subsection 3.1 to find the stationary equilibrium. Suppose that we have the guesses for sectoral capital and prices, $(K_j)_j$ and $(P_j)_j$. For sector j, plugging the expression of the inputs $(M_{ji})_i$ from Equation 4 in the expression of the input basket from Equation 2 gives:

$$M_j = \left(\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} ((1 - \alpha_j - \beta_j)\gamma_{ji} P_i^{-\sigma_j} \left(\sum_{k=1}^{N} \gamma_{jk} P_k^{1-\sigma_j} \right)^{-1} P_j Y_j))^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}}$$

$$M_j = (1 - \alpha_j - \beta_j) \left(\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} (\gamma_{ji} P_i^{-\sigma_j})^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}} \left(\sum_{k=1}^{N} \gamma_{jk} P_k^{1-\sigma_j} \right)^{-1} P_j Y_j$$

$$M_j = (1 - \alpha_j - \beta_j) \frac{P_j Y_j}{\tilde{P}_j}$$

Remember that labor supply is inelastic such that: $L_j = \tilde{L_j}\ \forall j$Plugging this expression and the guess of K_j in the production function Equation 1 yields:

$$Y_j = \xi_j Z_j K_j^{\alpha_j}\ L_j^{\beta_j} \left((1 - \alpha_j - \beta_j) \frac{P_j Y_j}{\tilde{P}_j} \right)^{1-\alpha_j-\beta_j}$$

such that we can compute Y_j with K_j, L_j and the prices at hand using the expression:

$$Y_j^{\alpha_j+\beta_j} = \xi_j Z_j K_j^{\alpha_j}\ L_j^{\beta_j} \left((1 - \alpha_j - \beta_j) \frac{P_j}{\tilde{P}_j} \right)^{1-\alpha_j-\beta_j}$$

All remaining quantities and prices are straightforward to compute with the expression of the $(Y_j)_j$ at hand.

C Distributional effects: graphs

The next figure plots the IRFs of Income Gini indices following a 1% negative supply shocks in the three sectors considered:

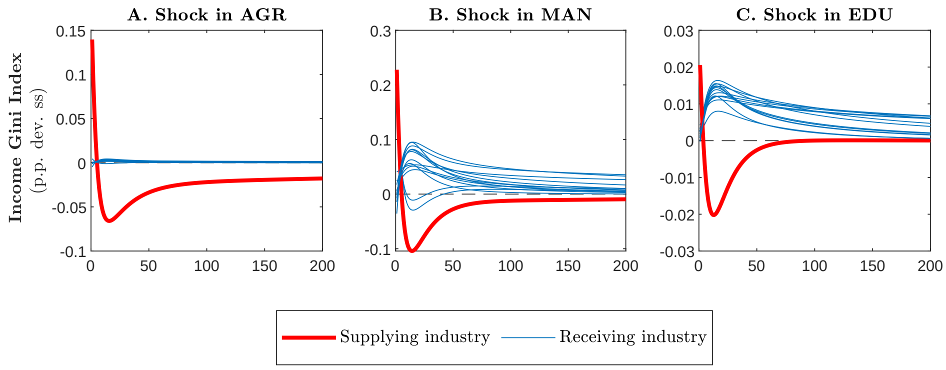


Figure 12: Income Gini index impulse responses

The next figure plots the IRFs of Wealth Gini indices following a 1% negative supply shocks in the three sectors considered:

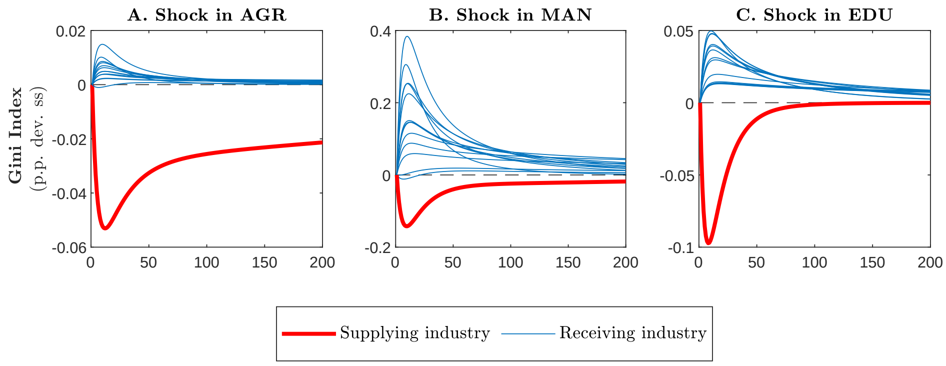


Figure 13: Wealth Gini index impulse responses

D Fiscal policy: additional graphs

D.1 Public-to-private ratios

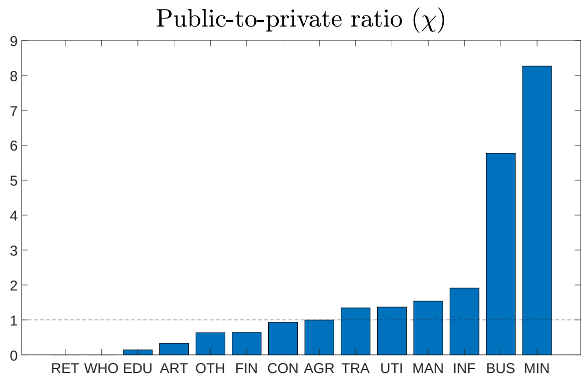
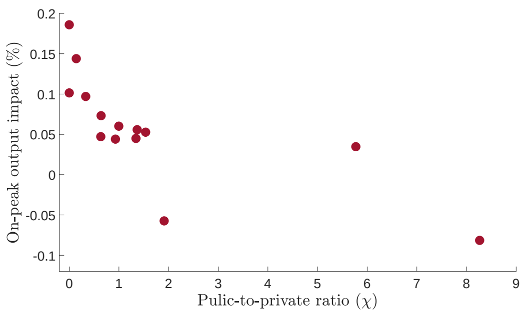
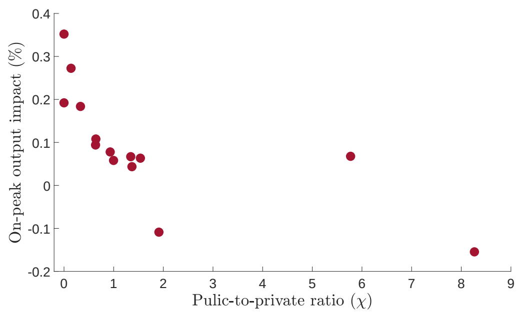


Figure 14: Public-to-private ratios

D.2 Sectoral output impacts



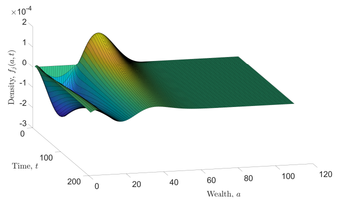
(a) Capital tax shock



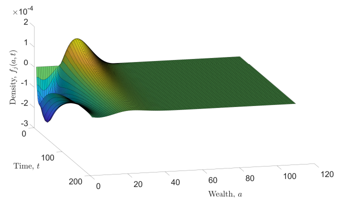
(b) Labor tax shock

Figure 15: Scatter-plots of on-peak output multipliers and Public-to-private ratio

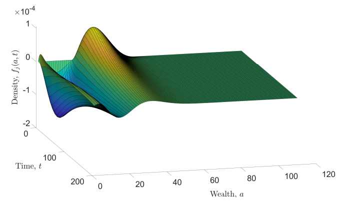
D.3 Distributional impacts



(a) Wealth in MIN

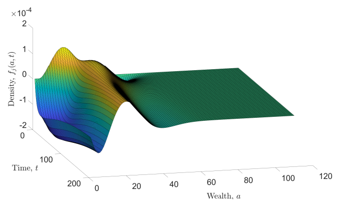


(b) Wealth in WHO

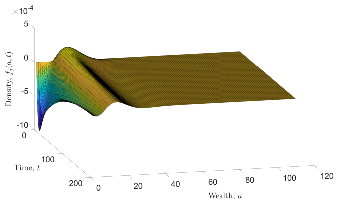


(c) Wealth in INF

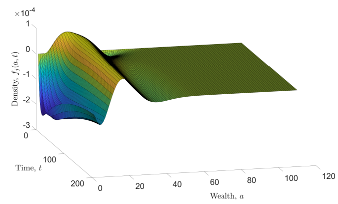
Figure 16: Evolution of income distribution over time for following a capital tax shock



(a) Wealth in MIN



(b) Wealth in WHO



(c) Wealth in INF

Figure 17: Evolution of income distribution over time for following a labor tax shock