Weitzman Meets Taylor: EU Allowances Futures Price Drivers and Carbon Cap Rules*

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Abstract

Using a two-sector DSGE model, we identify abatement, energy prices, transition demand for permits, and regulatory supply shocks as the key drivers of permit prices in the third phase of the EU Emission Trading System. We introduce an innovative approach to estimate the impact of abatement shocks by leveraging information from permit prices. The estimated price volatility is eighty times greater than it would be under a carbon price aligned with the Social Cost of Carbon. Our proposed rule-based cap adjustment, the Carbon Cap Rule, reduces volatility by 55% compared to the current EU ETS cap, and cuts welfare losses in consumption equivalence terms by half.

Keywords: EU ETS, Contingent Permit Allocation, Social Cost of Carbon, Bayesian Estimation, Carbon Central Bank.

JEL: Q58, G12, E32.

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1 Introduction

Cap-and-trade systems are increasingly favored as the primary mechanism for controlling emissions. While they enable the achievement of target emission reductions with minimal information requirements regarding regulated entities, they also introduce uncertainty around carbon prices due to factors such as economic activity, abatement costs, and policy decisions. This uncertainty can lead firms to delay decarbonization investments until financial returns become more predictable. Adjusting the cap based on available information could help, but the most influential factors remain unclear. We address this by developing a two-sector dynamic stochastic general equilibrium (DSGE) model to identify these factors and propose a novel approach to estimating less-observable shocks. Then, we introduce the "Carbon Cap Rule" (CCR), which dynamically adjusts the emission cap in response to key factors identified in the model. The CCR could reduce emission permit price volatility by about 55 percent and cut welfare losses by approximately 40 percent in CE terms compared to the optimal carbon pricing scenario, i.e. the social cost of carbon.

Most existing cap-and-trade systems function as "single-order" policies, where a fixed limit on pollution—the cap—is established (World Bank Group (2024)). While such systems are typically designed with specific institutional features, such as banking and price containment mechanisms, to provide some degree of flexibility in response to temporary shocks and to prevent extreme price fluctuations, carbon permit volatility in major markets has remained exceptionally high (Fuchs, Stroebel, and Terstegge (2024)). High volatility in carbon markets poses significant challenges. Carbon price uncertainty can severely impact firms' decisions to invest in decarbonization (Dixit and Pindyck (1994), Abel and Eberly (1996), Taschini (2021)). When permit prices are unpredictable, it becomes difficult for firms to plan and finance the development of low-carbon technologies, ultimately delaying the transition to a low-carbon economy. As lamented by European energy firms, "a carbon price floor would reduce volatility and uncertainty for any investor".

It has long been known that an ideal carbon pricing mechanism should be conditioned on the available information (Roberts and Spence (1976), Doda (2016a), Karp and Traeger (2023)). Instead of adhering to a fixed cap, the system could be designed as a contingency mechanism, where the cap adjusts dynamically in response to changes in economic conditions, technological advancements, and other relevant factors. This approach would aim to reduce volatility by aligning the supply of permits more closely with evolving demand, which adapts to the changing state of key factors, thereby creating a more predictable carbon pricing environment that supports long-term investments in emissions reduction.

A critical consideration is identifying the key factors to which the cap adjustment should be conditioned. The literature has explored relatively straightforward and observable indicators, like GDP and the permit price itself (Ellerman and Wing (2003) and Newell and Pizer (2008)). While indexing cap adjustments to these indicators can lead to welfare improvements compared to a fixed cap, the extent of these improvements depends heavily on the choice of indicators (Newell and Pizer (2008)).

Theory suggests that we should focus on the market fundamentals driving the demand for permits. The empirical literature identifies several key factors affecting permit demand: changes in goods consumption and production (Batten, Maddox, and Young (2021) and Friedrich, Mauer, Pahle, and Tietjen (2020)), energy prices (Friedrich et al. (2020)), companion policies for accelerating the low-carbon transition (referred to as transition demand) (Bjørnland, Cross, and Kapfhammer (2023)), and shifts in the availability and cost of abatement technologies (Newell, Jaffe, and Stavins (1999) and Karp and Traeger (2023)). Regulatory uncertainty has also been identified as a crucial driver of permit prices in the literature (Koch, Grosjean, Fuss, and Edenhofer (2016) and Känzig (2021)). As cap-and-trade systems are government-created, they are inherently susceptible to shifts in political leadership, public opinion, international agreements, and other external factors that can lead to changes in the regulatory framework. These changes can significantly alter the availability of permits, resulting in considerable fluctuations in permit prices.

To make sense of this empirical evidence and to understand the relevance of each factor – production, consumption, energy prices, transition demand, abatement, and permit supply – we utilize a two-sector DSGE model with a cap-and-trade framework. In this model, both the non-energy and energy sectors are subject to shocks in goods productivity and energy prices. Additionally, the energy sector faces uncertainty in abatement costs, while households encounter uncertainty in consumption. There is also uncertainty surrounding the carbon efficiency of energy decarbonization. Finally, we account for policy and regulatory uncertainties, which are reflected as supply shocks within the model.

While data on industrial production, consumption, and energy prices are readily available at a monthly frequency, abatement costs and transition demand for permits remain largely unobservable. To capture the "transition demand" dynamics – essentially changes in investors' sentiment regarding unexpected policy shifts that impact permit demand during the transition to a low-carbon economy – we utilize the climate sentiment index from Bua, Kapp, Ramella, and Rognone (2022). To capture abatement dynamics, we introduce an innovative method that takes advantage of the cap-and-trade system's design, which aims for a consistent reduction in emissions over time. By examining deviations from the expected emissions trend, attributable to supply shocks (i.e., regulatory changes), and integrating this analysis with permit price data, we can infer abatement cost shocks, since the prevailing permit price should equal the marginal cost of reducing an additional unit of emissions.

The model allows us to determine the relevance of each factor influencing carbon permit prices in the EU Emissions Trading System (ETS) between 2013 and 2019. Abatement costs, energy prices, transition demand, and permit supply are the primary drivers of changes in carbon permit prices during this period. As theory predicted, abatement emerges as the most significant driver, especially in the later stages of Phase 3 of the EU ETS. The strong correlation between the abatement factor and the EU's green investment further validates our model's interpretation of the importance of abatement for permit prices. The other three factors – energy price, transition demand, and permit supply – are nearly equally influential in driving permit prices. In contrast, total factor productivity (TFP) and consumption have a limited impact on carbon prices, highlighting their relatively minor role in carbon market dynamics.

To assess the extent of carbon price volatility within the EU ETS market, we first establish a baseline scenario where the carbon price aligns with the estimated Social Cost of Carbon (SCC). This baseline allows us to quantify the excess volatility present in the market, which we find to be approximately eighty times greater than the volatility observed under the SCC. Finally, we evaluate the costs associated with this excess volatility in terms of consumption equivalence. The welfare loss is approximately 0.006 percent in consumption equivalence terms when comparing the SCC scenario with the current EU ETS cap – a non-insignificant loss attributable to volatility in the carbon market.

Implementing the SCC is complex, and a more practical approach to reducing permit price volatility might involve devising a rule that regularly measures distance to the emission target while considering key factors, and then mechanically adjusting the cap in response to relevant shocks. To achieve this, we introduce a Carbon Cap Rule (CCR) designed to adjust the cap based on deviations from steady-state emissions and abatement costs, which are proven to be key driving factors. By applying the CCR, we obtain a substantial reduction in price volatility – approximately 55% compared to the volatility observed under the EU ETS cap. Moreover, the CCR cuts welfare costs in half, compared to the welfare losses in consumption equivalence observed under the EU ETS cap relative to the SCC.

Similar to how the Taylor rule guides central banks in adjusting interest rates in response to economic indicators – such as inflation and output gaps – the CCR provides a structured method for dynamically managing emission caps in response to critical environmental and economic factors. Therefore, the CCR has the potential to serve as a foundational rule for a Central Carbon Bank – a proposed institution for overseeing carbon market dynamics.

2 Carbon Prices and Uncertainty

Most existing cap-and-trade systems operate as "single-order" policies with a fixed limit on how much pollution can be emitted – the cap – and a rigid permit allocation schedule (World Bank Group (2024) and International Carbon Action Partnership (2024)). These systems are designed with specific institutional features intended to provide some flexibility in response to temporary shocks. Notably, banking and borrowing allow firms to manage their permits over time, offering temporal flexibility (Hasegawa and Salant (2015)). Additionally, cost and price containment mechanisms are implemented to prevent extreme fluctuations in permit prices.[1] However, despite these design provisions, carbon permit volatility in major markets has remained exceptionally high.[2] High volatility in carbon markets is problematic because it undermines the predictability of carbon pricing and creates an environment of uncertainty, which is detrimental to investment (Bernanke (1983), Dixit and Pindyck (1994), Abel and Eberly (1996), Bloom (2009), Taschini (2021)). When permit prices are unpredictable, firms and investors find it challenging to make substantial long-term commitments to abatement investments.[3] This uncertainty surrounding future permit prices makes it difficult to plan and finance the development of low-carbon technologies, ultimately delaying the transition to a low-carbon economy.[4]

[1]The EU ETS, the UK ETS, and the California cap-and-trade program allow firms to bank unused permits for future use, which provides some cushioning against price volatility. Additionally, each system includes measures like cost containment reserves and auction price floors to manage price extremes and prevent unexpected spikes in permit prices, though the specifics vary across programs (World Bank Group (2024) and International Carbon Action Partnership (2024)).

[2]This volatility often surpasses that of other commodity and financial markets. For example, permit prices in the EU ETS, UK ETS, and California are approximately 1.5, 1.24, and 1.3 times more volatile than Brent crude oil prices, respectively.

[3]In a recent article in the Financial Times, stakeholders expressed concern over this issue. "Europe's carbon price crash looks like serious market myopia", Financial Times February 28th 2024.

[4]For example, Clark, Bernstein, Beugin, Shaffer, and Wadland (2022) highlight that, in numerous discussions with industry leaders, business associations, commercial investors, and other stakeholders, the consistent message was that the uncertainty in carbon pricing is hindering investment and must be addressed urgently to accelerate industrial decarbonization. Similarly, in a joint statement by 16 European energy firms, one CEO emphasized that "a carbon price floor would reduce volatility and uncertainty for investors, making offshore wind projects without revenue-stabilizing mechanisms more viable, thereby accelerating the

It is widely understood that an ideal carbon pricing instrument should be conditioned on the available information (Weitzman (1974), Ellerman and Wing (2003), Newell and Pizer (2008), Doda (2016a), Karp and Traeger (2023)). Instead of sticking to a fixed cap, the system could be designed as a contingency mechanism, where the cap adjusts dynamically in response to changes in economic conditions, technological advancements, and other relevant factors (Roberts and Spence (1976)). This approach would aim to reduce volatility by aligning the supply of permits more closely with evolving demand, which adjusts to the changing state of key factors, thereby creating a more predictable carbon pricing environment that supports long-term investments in emissions reduction.

The key question then becomes: what are the critical factors to which the cap adjustment should be conditioned? Various factors have been proposed in the literature. For example, Ellerman and Wing (2003) and Newell and Pizer (2008) suggest conditioning the cap on domestic GDP, allowing the cap to adjust to shocks in the economic activity. Alternatively, Burtraw, Holt, Palmer, and Shobe (2020) propose conditioning the current permit allocation on previous prices. This approach uses past permit price fluctuations as a signal to adjust the cap, aiming to reduce volatility. These studies conclude that adaptive cap policies, which adjust based on relevant indicators, can lead to welfare improvements compared to a fixed cap. However, the extent of these improvements depends significantly on the choice of indicators. While GDP and past prices are relatively straightforward options, they may not fully capture the most critical factors influencing the demand and supply of permits.

Building on the theoretical foundation that the price of emission permits responds to shocks in both the stringency of the system –namely, the *supply* of permits– and the market fundamentals associated with the *demand* of permits, we now turn to identify the key factors that influence both supply and demand, drawing on empirical evidence from the literature.[5]

The empirical literature has consistently documented that economic activity is a key driver of permit prices. For example, Batten et al. (2021) and Friedrich et al. (2020) highlight the strong correlation between economic performance and the demand for emission permits. This relationship is largely because economic activity drives goods consumption and production, which in turn affects the volume of emissions that firms must either reduce or offset by purchasing permits. Conceptually, changes in goods consumption and production can be directly linked to shifts in business-as-usual emissions, which can be described

critical transition to low-carbon energy systems" (EnBW (2018)).

[5]For a recent survey of permit pricing theory, see Weitzman (1974), Hoel and Karp (2002), Newell and Pizer (2003), Hasegawa and Salant (2015).

through *total-factor productivity (TFP)* shocks and *consumption* shocks. These shocks reflect how efficiently the economy uses its inputs, such as labor and capital, and how consumer demand changes over time.

In addition to economic activity, *energy prices* of energy consumed are significant drivers of permit prices (Friedrich et al. (2020) and references therein). Higher energy prices typically increase the cost of production, prompting firms to either reduce emissions or purchase more permits, thereby driving up permit prices. Conversely, lower energy prices can reduce the demand for permits as firms find it less costly to operate without exceeding their emission limits.[6]

In addition to economic activity and energy prices, companion policies such as those supporting renewable energy generation and energy efficiency also play a significant role in shaping the demand for emission permits. These policies are designed to accelerate the transition to a low-carbon economy by encouraging the adoption of cleaner energy sources and reducing overall energy intensity. As firms become more energy-efficient, the need for energy-intensive production processes decreases, leading to lower emissions. This reduction in energy demand directly impacts the carbon market by reducing the volume of emissions that need to be offset, thereby decreasing the demand for permits (Bjørnland et al. (2023)). However, an unexpected reversal or weakening of these supportive policies can create significant shocks in permit *transition demand*. For instance, if a government rolls back subsidies for renewable energy or relaxes energy efficiency standards, the anticipated decrease in energy demand may not materialize, or it may reverse, leading to higher emissions. This sudden change can catch the market off guard, leading to shocks in the demand for permits as firms find themselves needing to purchase more permits than previously expected to cover their higher-than-anticipated emissions.

Theoretical models emphasize the role of *abatement* supply as a key driver of emission permit demand (Rubin (1996), Newell and Pizer (2003), Schennach (2000), Kollenberg and Taschini (2016), Karp and Traeger (2023)). Shifts in abatement supply –essentially the availability and cost of technologies that reduce emissions– are crucial in determining permit demand. Abatement costs, which represent the expenses associated with reducing emissions, directly influence how much firms are willing to pay for permits. Unexpected changes in the availability and prices of abatement technologies can lead to significant shocks in abatement

[6] Borenstein, Bushnell, Wolak, and Zaragoza-Watkins (2019) documents these factors as critical determinants of permit prices in the California cap-and-trade program. Studies such as Hintermann, Peterson, and Rickels (2016) and Friedrich et al. (2020) further validate the importance of energy prices in the context of the EU ETS.

costs, which in turn can alter the demand for emission permits

Figure ?? highlights specific events in the EU ETS system attributed to (i) productivity and consumption shocks, (ii) energy shocks, (iii) transition demand shocks, and (iv) abatement shocks, illustrating how these factors have significantly impacted permit prices.

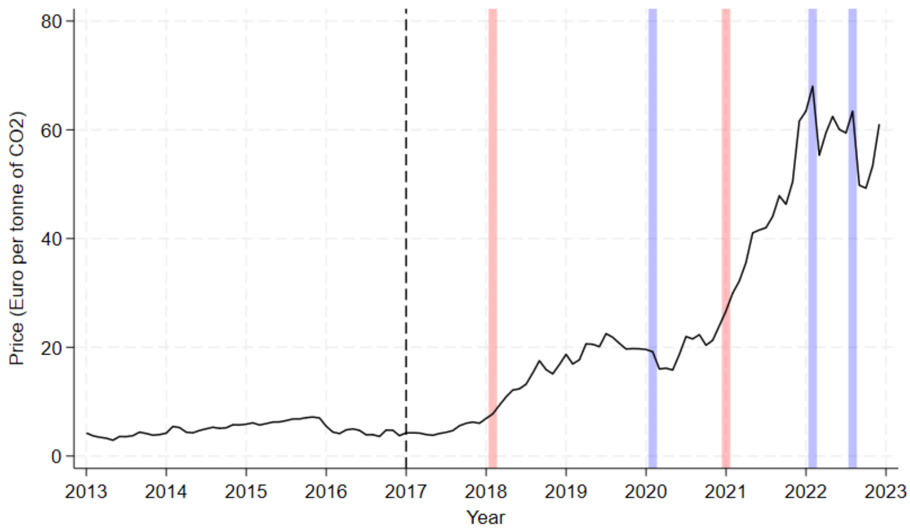


Figure 1: Real EU ETS price - demand vs supply shocks

Note: Real EU ETS price at monthly frequency from January 2013 to December 2022. Red bars indicate supply shocks and correspond to EU ETS Phase 4 approval (Feb. 2018) and Phase 4 start (Jan. 2021). Blue bars indicate demand shocks and correspond to COVID-19 onset (February 2020), Ukraine invasion (February 2022), and ECB interest rate hike over a decade (August 2022). Source: Refinitiv Eikon.

Figure ?? also illustrates relevant policy events that have influenced the *supply* of permits. These events often reflect regulatory changes, such as adjustments to the cap or permit allocation. Regulatory uncertainty has been identified as a crucial driver of permit prices in the literature (Koch et al. (2016); Deeney, Cummins, Dowling, and Smeaton (2016); Känzig (2021)). Since cap-and-trade systems are government-created, they are inherently susceptible to shifts in political leadership, public opinion, international agreements, and other external factors that can lead to changes in the regulatory framework. Such shifts introduce uncertainty into the market, as participants are unable to accurately predict how the supply of permits will be adjusted in the future. This uncertainty can result in significant price fluctuations as market participants react to potential changes in the availability of permits.

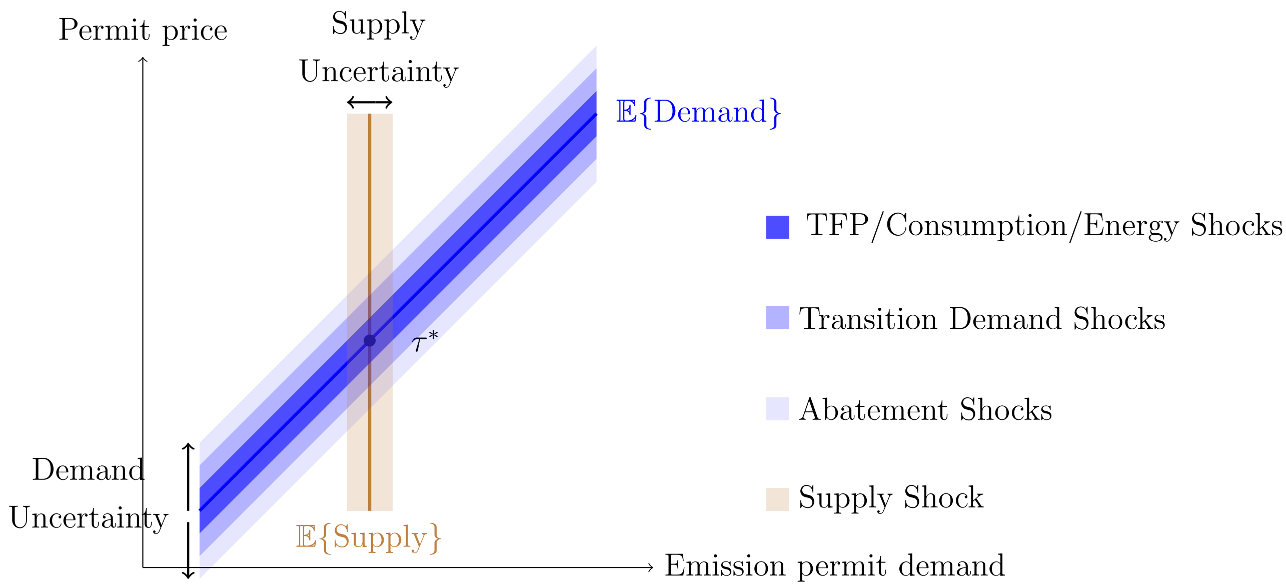


Figure 2: Demand and Supply Shocks To Price

The combined shocks in TFP, consumption, energy prices, transition demand, and abatement all contribute to overall demand shocks in the carbon market, as illustrated in Figure 2. When combined with supply shocks, they can introduce significant market volatility. In the context of contingent carbon pricing and adaptable cap setting, it is essential to understand the relevance and unpredictability (as illustrated by the width of the shaded areas) of these demand and supply factors. In the remainder of the paper, we evaluate the relevance of these shocks and explore how a more adaptive cap-and-trade system could mitigate their effects.

3 The model

In this section, we rationalize the findings of the empirical literature using a two-sector DSGE model with climate policies. First, we outline the model's features, followed by a description of each element and the role of the environmental regulator.

We consider an infinite-horizon, closed economy with two production sectors (energy producers and final firms), households, a government, and an environmental regulator. Households are identical, infinitely lived, and collectively measure one. Energy producers generate CO_2 emissions, creating an environmental externality that raises temperatures and impacts household welfare. However, energy producers do not internalize the environmental consequences of their emissions, leading to market failure. The rest of this section outlines the model in more detail.

3.1 *Climate change and emission dynamics*

Building on standard integrated assessment models (IAMs) (see Nordhaus (1991) and Nordhaus and Yang (1996)), we model the relationship between CO_2 emissions and global temperature (T_t^o), assuming a linear relationship where the global temperature is directly proportional to the cumulative CO_2 emission stock – as posited by Matthews, Gillett, Stott, and Zickfeld (2009):

$$T_{t+1}^o = \zeta_1^o(\zeta_2^o X_t - T_t^o) + T_t^o, \tag{1}$$

with ζ_1^o and ζ_2^o chosen following Dietz and Venmans (2019).[7]

Cumulative CO_2 emission denoted as X_t, evolves according to the following law of motion:[8]

$$X_{t+1} = \eta X_t + E_t^T + E_t^{RW}, \tag{2}$$

where X_{t+1} is the concentration of CO_2 in the atmosphere; anthropogenic CO_2 emissions consist of both energy and non-energy sources, with total inflow at time t denoted as $E_t^T := E_t^E + E_t^{\mathrm{NE}} \geq 0$ from the energy (E_t^E) and non-energy (E_t^{NE}) sectors within the Euro Area. The term E_t^{RW} represents emissions from the rest of the world. The parameter $0 < \eta < 1$ represents the persistence of CO_2 emissions, typically set close to 1, as argued by Matthews et al. (2009).

Emissions from the energy sector E_t^E result from energy production Y_t^E and are influenced by an exogenous trend Γ_t^X, which captures the decoupling between CO_2 emissions and energy production. The relationship can be expressed as:

$$E_t^E = (1 - \mu_t)\, \varphi_{\mathrm{E}} \epsilon_t^{\varphi_{\mathrm{E}}} Y_t^E \Gamma_t^X, \tag{3}$$

where the variable $0 \leq \mu_t \leq 1$ represents the fraction of emissions mitigated (abated) by energy firms, while $\varphi_{\mathrm{E}} \geq 0$ is the carbon-intensity parameter defining the steady-state

[7] We observe that although variations in climate dynamics and damage modeling over the long horizon (be it à la Golosov, Hassler, Krusell, and Tsyvinski (2014), à la Nordhaus (2017), or à la Matthews et al. (2009), among others) lead to subsequent effects on macroeconomic aggregate equilibria, over the business cycle horizon (and under equivalent calibrations), these modeling specifications do not result in significant changes to macroeconomic aggregate equilibria.

[8] To ensure convergence in the auto-regressive law of motion for the stock of emissions process, and without a loss of generality, we deviate slightly from the transient climate response to cumulative CO_2 emissions theory by setting $\eta \neq 1$. However, we select η to be sufficiently close to one so that $X_t \approx X_0 + \sum_{i=0}^{t}(E_i^T + E_i^{RW})$.

relationship between emissions and energy output. The product $\varphi_{\mathrm{E}} Y_t^E$ represents the total CO_2 influx from production before any abatement measures. The shock to the carbon intensity of energy production, $\epsilon_t^{\varphi_{\mathrm{E}}}$, captures fluctuations in the carbon efficiency of energy decarbonization, which we associate with transition demand shocks, and follows an AR(1) process:[9]

$$\log\left(\varepsilon_t^{\varphi_{\mathrm{E}}}\right) = \rho_{\varphi_{\mathrm{E}}} \log\left(\varepsilon_{t-1}^{\varphi_{\mathrm{E}}}\right) + \eta_t^{\varphi_{\mathrm{E}}}, \text{ with } \eta_t^{\varphi_{\mathrm{E}}} \sim N(0, \sigma_{\varphi_{\mathrm{E}}}^2).$$

CO_2 emissions from the non-energy sector follow a similar pattern to those from the energy sector, but with a key difference: the non-energy sector is not subject to environmental policies or carbon pricing. As a result, firms in this sector do not engage in emissions abatement:

$$E_t^{\mathrm{NE}} = \varphi_{\mathrm{NE}} Y_t^{\mathrm{NE}} \Gamma_t^X \tag{4}$$

with φ_{NE} representing the emission intensity in the non-energy sector.

3.2 Energy Firms

Energy producers aim to maximize profit by optimizing their levels of capital and labor while considering energy prices, abatement costs, and policy costs. Energy production is modeled using a Cobb-Douglas production function:

$$\tilde{Y}_t^E = \varepsilon_t^{A_E} A_t^E (K_t^E)^{\alpha_E} (\Gamma_t^Y l_t^E)^{1-\alpha_E} \Gamma_t^{Y^E}, \tag{5}$$

where K_t^E represents the capital stock used by the energy firms with an intensity parameter $\alpha_E \in [0,1]$, l_t^E denotes labor, $A_t^E > 0$ denotes the productivity level, and $\varepsilon_t^{A^E}$ is a total energy productivity shock that evolves as follows:

$$\log\left(\varepsilon_t^{A^E}\right) = \rho_{A^E} \log\left(\varepsilon_{t-1}^{A^E}\right) + \eta_t^{A^E}, \text{ with } \eta_t^{A^E} \sim N(0, \sigma_{A^E}^2).$$

To achieve a more accurate representation of energy production within the model, we account for the differing growth dynamics between the energy sector and the broader economy. Specifically, we incorporate an energy transition trend $\Gamma_t^{Y^E} = \gamma^{y^E} \Gamma_{t-1}^{Y^E}$, which captures the growth rate specific to the EU's energy and industrial production sectors. This approach allows us to model the distinct trajectory of the energy sector as it undergoes transformation

[9]The shock to the carbon intensity of energy production represents unforeseen changes in the energy sector's decarbonization efforts. We associate these fluctuations with transition demand shocks, indicating deviations from the anticipated policy-driven pace of the shift to low-carbon energy generation.

in line with policy goals and technological advancements. Consequently, the trend-corrected energy production is expressed as $Y_t^E = \tilde{Y}_t^E \Gamma_t^{Y^E-1}$. The implications of this trend correction are further discussed in the Balanced Growth Path (BGP) section.

Energy producers maximize profits:

$$\Pi_t^E = \varepsilon_t^p p_t^E Y_t^E - w_t^E l_t^E - I_t^E - Z_t - \tau_t E_t^E. \tag{6}$$

The relative price of energy and the real wage are denoted by p_t^E and w_t^E, respectively. The shock to energy price, ε_t^p, evolves as an AR(1):

$$\log\left(\varepsilon_t^p\right) = \rho_p \log\left(\varepsilon_{t-1}^p\right) + \eta_t^p, \text{ with } \eta_t^p \sim N(0, \sigma_p^2).$$

The abatement-cost function per unit of energy production is represented by $Z_t = f\left(\mu_t\right) Y_t^E$. Additionally, $\tau_t \geq 0$ is a the carbon price set by the environmental regulatory authority, which will be detailed later. Investment is denoted by I_t^E, and the accumulation of physical capital follows the law of motion:

$$K_{t+1}^E = (1-\delta)K_t^E + I_t^E, \tag{7}$$

where $\delta \in [0, 1]$ is the depreciation rate of physical capital.

The abatement-cost function is adapted from Nordhaus (2008) and is defined as $f\left(\mu_t\right) = \theta_1 \mu_t^{\theta_2} \varepsilon_t^z$. In this expression, $\theta_1 \geq 0$ determines the steady state of the abatement, while $\theta_2 > 0$ represents the elasticity of the abatement cost with respect to the fraction of emissions abated. This function links the fraction of emissions abated to the portion of output allocated to abatement, with the abatement price normalized to one. The abatement shock, ε_t^z, which captures market uncertainties related to abatement costs and technology, evolves as follows:

$$\log\left(\varepsilon_t^z\right) = \rho_z \log\left(\varepsilon_{t-1}^z\right) + \eta_t^z \text{ with } \eta_t^z \sim N(0, \sigma_z^2).$$

3.3 Final goods firms

Final firms aim to maximize profit by optimizing their levels of capital and labor while considering energy prices. Following Bachmann, Baqaee, Bayer, Kuhn, Löschel, Moll, Peichl, Pittel, and Schularick (2022), firms operate under a Constant Elasticity of Substitution

(CES) production function,[10] capturing the non-linear dynamics of substituting away from energy to other production inputs.

$$Y_t = \left((1-\chi)^{\frac{1}{\sigma}} (Y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} Y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{8}$$

with

$$Y_t^{\mathrm{NE}} = \varepsilon_t^{A^{\mathrm{NE}}} A_t^{\mathrm{NE}} (K_t^{\mathrm{NE}})^{\alpha_{\mathrm{NE}}} (\Gamma_t^Y l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{9}$$

where K_t^{NE} is the capital stock utilized by final firms with an intensity parameter $\alpha_{\mathrm{NE}} \in [0,1]$, l_t^{NE} is non-energy labor, $A_t^{\mathrm{NE}} > 0$ is the productivity level of the non-energy final sector, σ is the elasticity of substitution between the energy and non-energy production factors, χ is the energy share in total production, Γ_t^{YE} is the exogenous corrective trend for the energy sector to match EU growth dynamics while maintaining a BGP, and $\varepsilon_t^{A^{\mathrm{NE}}}$ is a total factor productivity shock that evolves as follows:

$$\log\left(\varepsilon_t^{A^{\mathrm{NE}}}\right) = \rho_{A^{\mathrm{NE}}} \log\left(\varepsilon_{t-1}^{A^{\mathrm{NE}}}\right) + \eta_t^{A^{\mathrm{NE}}} \quad \text{with } \eta_t^{A^{\mathrm{NE}}} \sim N(0, \sigma^2_{A^{\mathrm{NE}}})$$

While much of the climate economics literature models environmental damages on the production side, following Nordhaus (1991), we take a different approach by incorporating these damages within the utility function of households, as done by Barnett, Brock, and Hansen (2020). This approach, though potentially isomorphic to production damages under certain functional forms and calibration, allows us to maintain a balanced growth path without imposing restrictive assumptions on the damage function and its parameters. We explore this in more detail in the Balanced Growth Path section. Additionally, in Appendix C we show that modeling damages through utility or production yields similar results.

Final firms maximize profits:

$$\Pi_t^F = Y_t - w_t^{\mathrm{NE}} l_t^{\mathrm{NE}} - I_t^{\mathrm{NE}} - \varepsilon_t^p p_t^{\mathrm{E}} Y_t^{\mathrm{E}}. \tag{10}$$

The real wage is denoted by w_t^{NE}, and capital investment by I_t^{NE}. The accumulation of physical capital follows a similar law of motion as in the energy sector:

$$K_{t+1}^{\mathrm{NE}} = (1-\delta) K_t^{\mathrm{NE}} + I_t^{\mathrm{NE}}, \tag{11}$$

[10]In Appendix D for robustness, we also consider the simpler Cobb-Douglas aggregator: $Y_t = \varepsilon_t^{A_y} A_t^y (K_t^{\mathrm{NE}})^{\alpha_{\mathrm{NE}}} (Y_t^E \Gamma_t^{E^{-1}})^{\alpha_{\mathrm{E}}} (\Gamma_t^Y l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}-\alpha_{\mathrm{E}}}$, where α_{E} represents the energy share and α_{NE} the non-energy capital share.

where $\delta \in [0, 1]$ is the depreciation rate of physical capital.

3.4 Households

Households make consumption and investment (savings) decisions, and supply labor inelastically. They hold government bonds and own firms in the corporate sector, receiving dividends or profits. They also face climate damages, denoted as $D_u(T_t^o)$, representing disutility from rising temperatures, following the approaches of Barnett et al. (2020) and Barrage (2020). Households maximize their life-time utility:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B u\left(C_t - H_{t-1} - D_u(T_t^o)\right), \tag{12}$$

where $\mathbb{E}_t$ is the expectations operator conditioned on information at time t, β is the time discount factor, C_t represents consumption, H_{t-1} represents consumption habits, and ε_t^B is the preference shock

$$\log \varepsilon_t^B = \rho_B \log \varepsilon_{t-1}^B + \eta_t^B \quad \text{with } \eta_t^B \sim N(0, \sigma_B^2)$$

Climate damages are linear in temperature:

$$D_u(T_t^o) = \Theta_t^T T_t^o$$

where Θ_t^T represents households' sensitivity to temperature increases.

The habit stock's law of motion follows Campbell and Cochrane (1999), with $H_{t-1} = hC_{t-1}$. Unlike Cai and Lontzek (2019), who use recursive utility à la Epstein and Zin (1989) to capture long-run risk we choose consumption habits for two reasons: (i) our focus is on the business cycle, specifically phase 3 of the EU ETS from 2013 to 2019, where long-run climate risk is less relevant, and (ii) consumption habits are crucial for generating higher volatility in the social cost of carbon during business cycle fluctuations, as shown by Benmir, Jaccard, and Vermandel (2020), while still aligning with observed consumption and output volatility in real data.

The budget constraint of the representative household is:

$$w_t^{\mathrm{NE}} l_t^{\mathrm{NE}} + w_t^{\mathrm{E}} l_t^{\mathrm{E}} + r_t B_t + \Pi_t^E + \Pi_t^F - T_t = C_t + B_{t+1} \tag{13}$$

where the left-hand side represents the household's income sources, primarily labor earnings. The household also earns returns from holding long-term government bonds, denoted as B_t, with a return rate of r_t. As owners of firms in the corporate sector, they receive dividend income from both energy firms (Π_t^E) and final firms (Π_t^F). On the spending side, the household allocates income to consumption goods (C_t) and the purchase of long-term government bonds (B_t). Additionally, the government imposes a lump-sum tax, denoted by T_t.

3.5 Government and market clearing

The government funds its expenditures through tax collection, with its budget constraint given by:

$$G_t = T_t + \tau_t E_t, \tag{14}$$

where G_t represents public expenditure and T_t is a lump-sum tax. The second revenue component, $\tau_t E_t$, represents earnings from imposition a cost on environmental externalities. Here, E_t denotes emissions, and τ_t is the carbon price – the cost of emitting one unit of CO_2 emission.

As is standard in most business-cycle models, government expenditure is a proportion of the total output. The economy's resource constraint is expressed as:

$$Y_t = C_t + I_t^{\mathrm{NE}} + I_t^{\mathrm{E}} + G_t + Z_t. \tag{15}$$

3.6 Emission cap

The environmental regulator sets the cap Q_t independently of the associated climate damage as it is the case in Europe within the ETS market. In theory, the regulator would set the cap optimally by equating the marginal costs to the marginal benefits of emission reduction, reflecting the social cost of carbon that a social planner would choose in a centralized economy.[11]

In practice, however, setting the cap is a complex political process that balances environmental goals with socio-economic considerations. As societal priorities change, policymakers may adjust emission limits to reflect these evolving conditions. Such adjustments can introduce policy and regulatory uncertainties which are reflected as supply shocks. In line with our empirical observations, shifts in climate policies are modeled as exogenous changes in

[11] The optimal cap and corresponding carbon price are calculated in Appendix A.

the cap:

$$E_t^E = Q_t \varepsilon_t^S, \tag{16}$$

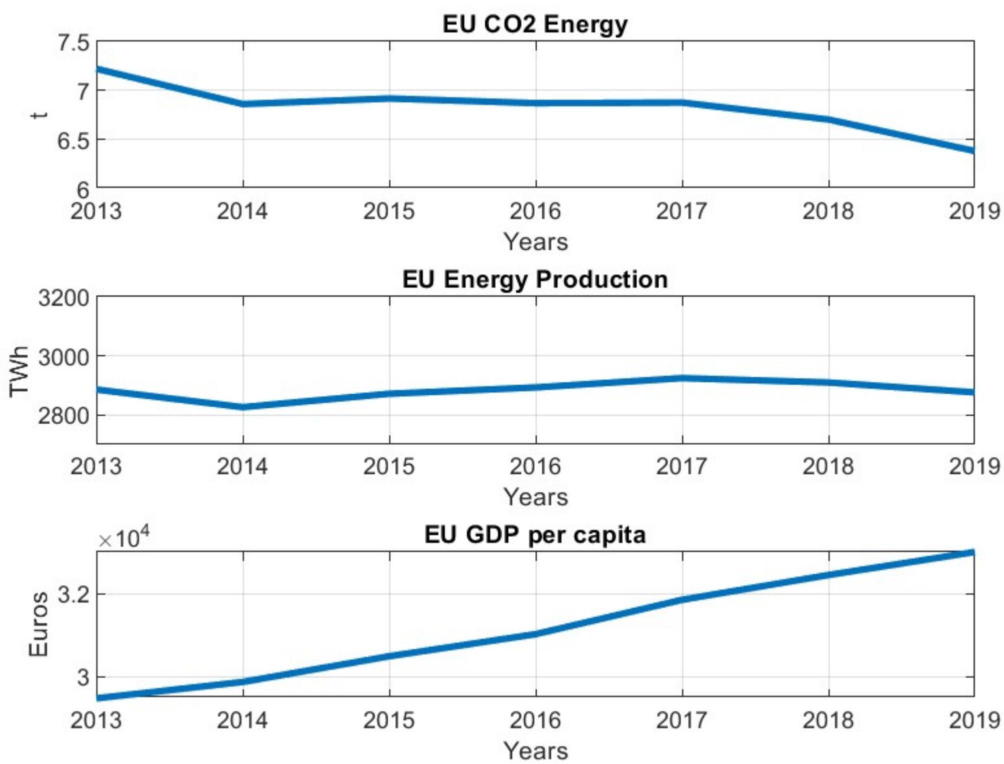
where ε_t^S evolves as:

$$\log \varepsilon_t^S = \rho_S \log \varepsilon_{t-1}^S + \eta_t^S \quad \text{with } \eta_t^S \sim N(0, \sigma_S^2).$$

3.7 Balanced growth

As illustrated in Figure 3, the EU's energy emissions, energy production, and overall output exhibit distinct growth rates.

Figure 3: EU Trends in CO2 Energy Emissions, Energy Production (Electricity), and GDP Per Capita



Notes: The figure was generated utilizing data on CO2 emissions and energy production from https://ourworldindata.org/, along with GDP per capita data sourced from FRED.

Specifically, we observe that while the output of the economy grows over time, the energy sector's growth rate is stationary, reflecting the zero growth rate observed in the data (i.e.,

$\Gamma_t^E = \Gamma_t^{Y^{-1}}$). This stationary behavior contrasts with the growth observed in other sectors of the economy, particularly in overall output. The disparity in growth rates between energy emissions and output can be largely attributed to the introduction of green technological progress. As green technologies advance, they enable more efficient production processes that reduce emissions, even as economic output continues to grow. This divergence highlights the importance of considering technological progress in any analysis of energy production and emissions.

Given that the primary focus of this paper is to estimate the drivers of carbon permit prices, it is crucial to properly account for different growth dynamics within our model. To this end, we derive a de-trended model along its balanced growth path. This approach allows us to account for the varying growth rates across different sectors, particularly in cases where emissions and energy production do not align with the overall output growth.

Consistent with the literature, we assume that macroeconomic variables grow along the balanced growth path, facilitated by labor-augmenting technological progress, represented by Γ_t^Y. The growth rate of this technological progress is denoted by γ^y, and is defined by the relationship:

$$\frac{\Gamma_{t+1}^Y}{\Gamma_t^Y} = \gamma^y.$$

In our model, we also account for green technological progress, which plays a critical role in the decoupling of economic output growth from emission growth. We represent this progress by Γ_t^X, with its growth rate, γ^x, defined as:

$$\frac{\Gamma_{t+1}^X}{\Gamma_t^X} = \gamma^x.$$

This trend is essential for capturing the long-term shift toward less energy-intensive production processes, which is a key factor in reducing emissions while maintaining economic growth. As documented by Newell et al. (1999), this energy-saving technological progress can be interpreted as the adoption of more efficient, less carbon-intensive technologies, particularly in capital goods. An improvement in this green technology is reflected by a value for γ^x that is below 1, indicating a reduction in energy intensity over time.

In Appendix E, we introduce the de-trended economy, providing a detailed derivation of the model. We also discuss the social planner problem and the decentralized problem in detail, offering insights into how these growth dynamics influence policy decisions and market outcomes in the context of carbon pricing and emissions regulation.

4 Bringing the Model to Data

Bringing our model to data is essential to disentangle the drivers of the EUA futures. A major challenge, however, stems from the unobservable nature of abatement dynamics at the monthly frequency. To overcome this limitation and fill the data gap in these areas, we have devised an innovative methodology for estimating the shocks associated with abatement costs. We also pay particular attention to match a broad spectrum of statistics, including the share of EU emissions, emissions per sector within the EU, and the energy intensity of each sector.

4.1 Data

We assembled our dataset by integrating multiple sources. Macroeconomic data such as industrial production survey (i.e. Production Index Growth), consumer confidence survey (i.e. Consumption Index Growth), energy production (i.e. Per Capita Energy Production Growth), and pricing information (i.e. Energy Production Price) are taken from EUROSTAT. Carbon dioxide emissions data (i.e. Per Capita Emissions Growth) are taken from EDGAR.[12] The climate sentiment index which captures the "Transition Demand" dynamics[13] is taken from Bua et al. (2022).[14]

Finally, data on European Union Allowance (EUA) futures prices from the Intercontinental Exchange (ICE) are taken from Bloomberg.[15] We consider data for the third phase of the European Union Emission Trading System (EU ETS), corresponding to January 2013 to December 2019, and restrict the data to countries within the European Union Emission Trading System (EU ETS) framework, considering a total of 28 countries for our analysis (including the UK).[16] A complete description of each data source used is in Appendix A.

[12](See Commission, Centre, Crippa, Guizzardi, Schaaf, Monforti-Ferrario, Quadrelli, Risquez Martin, Rossi, Vignati, Muntean, Brandao De Melo, Oom, Pagani, Banja, Taghavi-Moharamli, Köykkä, Grassi, Branco, and San-Miguel (2023).

[13]Climate sentiment are data allows for to capturing investors sentiment and commitment toward net-zero transition.

[14]For capturing changes in expectations regarding the EU's transition to a low-carbon economy, we use the climate sentiment index (i.e., Transition Demand).

[15]One EUA grants permission to emit one tonne of carbon dioxide (CO_2) or an equivalent amount of another greenhouse gas. "EUA Futures" refers to futures contracts based on these allowances.

[16]The EU ETS currently operates in 30 countries: the 27 EU member states plus Iceland, Liechtenstein, and Norway. The United Kingdom left the EU on 31 January 2020 but remained subject to EU rules until 31 December 2020. Due to data constraints, we omit Norway and Liechtenstein.

4.2 Strategy

We estimate our model using Bayesian methods on monthly EU data from January 2013 to December 2019. To map our model to the data, we augment our equilibrium equations with observation equations as follows:

$$\begin{bmatrix} \text{Production Index Growth} \\ \text{Consumption Index Growth} \\ \text{Per Capita Emissions Growth} \\ \text{Per Capita Energy Production Growth} \\ \text{Energy Production Price} \\ \text{Real } CO_2 \text{ Price Growth} \\ \text{Transition Demand} \end{bmatrix} = \begin{bmatrix} (\gamma^y y_t - y_{t-1})/y_{t-1} \\ (\gamma^y c_t - c_{t-1})/c_{t-1} \\ \log \gamma^s + \Delta \log (e_t) \\ \Delta \log \left(y_t^E\right) \\ \Delta \log \left(p_t^E\right) \\ \Delta \log (\tau_t) \\ \Delta \log (\mu_t) \end{bmatrix}, \tag{17}$$

where γ^s represents the trend in emissions and γ^y denotes the trend growth rate of the economy.[17] Considering the model's stationary nature, it is imperative to transform the data series into a stationary form before integrating them into the model. In line with the foundational approach established by Smets and Wouters (2007), we address data that exhibit a unit root by rendering them stationary. This is achieved by taking the logarithmic difference of the series as necessary.

Data availability at monthly frequency

We can reliably use series for energy prices, energy supply, and CO2 prices from Eurostat and Bloomberg. However, obtaining high-quality data for other observables that we want to incorporate into our model can be more challenging. Specifically, we rely on surveys for production and consumption data. Recognizing that these series may contain inaccuracies, we include measurement errors in our estimation procedure.

For emissions, we use the EDGAR dataset, which provides high-frequency, highly disaggregated emissions data by sector. We also account for the fact that this data is produced using interpolation techniques by including a measurement error.

Regarding climate sentiment (as provided by Bua et al. (2022)), we assume its variations are a good proxy for the willingness of firms to abate, as it captures shifts in transition risk within the EU which we refer to as transition demand. Therefore, we map this sentiment to

[17] We refer to Appendix E for the full description of the BGP.

the business cycle variations of abatement in our economy.

Inferring the abatement cost series

Our model specification, along with the specified set of observables, allows us to disentangle the various supply and demand dynamics at play for firms subject to carbon pricing (discussed in section 2). More specifically, we leverage the intrinsic design of the cap-and-trade system, which is structured to achieve a consistent reduction in emissions over the duration of our study. This system's design is crucial because it provides a framework where emission targets are incrementally tightened, indicative of a progressively decreasing per-period cap. Therefore, any deviation in emissions from the trend can be attributed to supply shocks (i.e., regulation). This could also capture firms' compliance with the targets set by the regulator. By incorporating the emission allowances price series, we can infer abatement cost shocks, given that marginal costs and the price of emissions are equated in the first-order condition of firms.

4.3 Calibration

We summarise in this section the parametrisation of the model. For parameters for which the time interval is relevant, the calibration is monthly. Consistent with standard practice, we have tailored the model's calibration to align with certain observed key aggregates. These include temperature, the share of EU emissions, emissions per sector within the EU, the energy intensity of each sector, and the average value of the EU ETS allowance price, all specifically within the context of the European Union. This calibration ensures that our model accurately reflects the real-world dynamics and trends of these critical environmental and economic indicators.

The parameters pertaining to the business cycle structure of our model are conventional. For the standard parameters in these models, such as the discount factor β and the risk aversion σ^U, we align to typical values used in macroeconomic modeling.[18] Specifically, the capital intensity parameters are set at $\alpha_N = \alpha_{NE} = 0.333$, while the depreciation rate δ is fixed at 0.008. The discount factor β is set at 0.9986 and the risk aversion σ^U at 1.5.

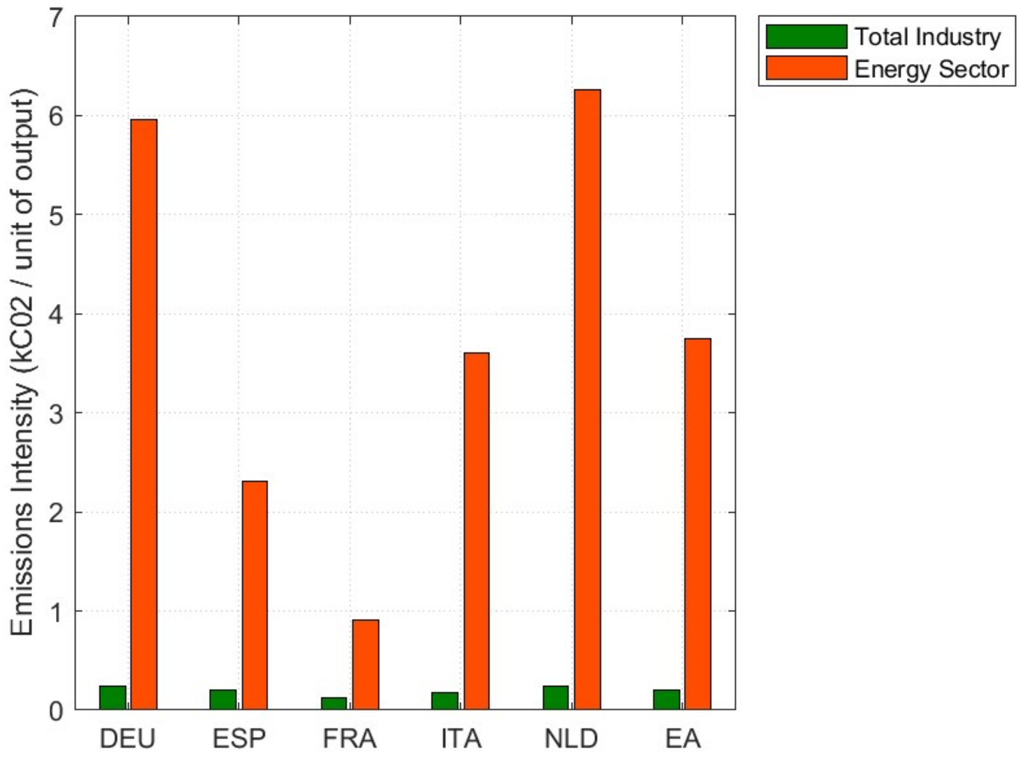
In calibrating the climate block of the model, we follow Dietz and Venmans (2019) and set the parameters for the global temperature function $\zeta_1^o = 0.50$ and $\zeta_2^o = 0.00125$.

We use the remaining parameters to match a number of relevant statistics for the EU.

[18] Notice that we calibrated all the parameters to a monthly frequency.

Specifically, the share of the energy sector in the economy χ is fixed at 2%, while the elasticity in the CES function σ is 0.2, consistent with estimates in Labandeira, Labeaga, and López-Otero (2017). The emission intensity parameters φ_E and φ_{NE} are calibrated to match emission to production in both sectors. As depicted in Figure 4, the energy sector is approximately thirty times more emission intensive than the industry as a whole.

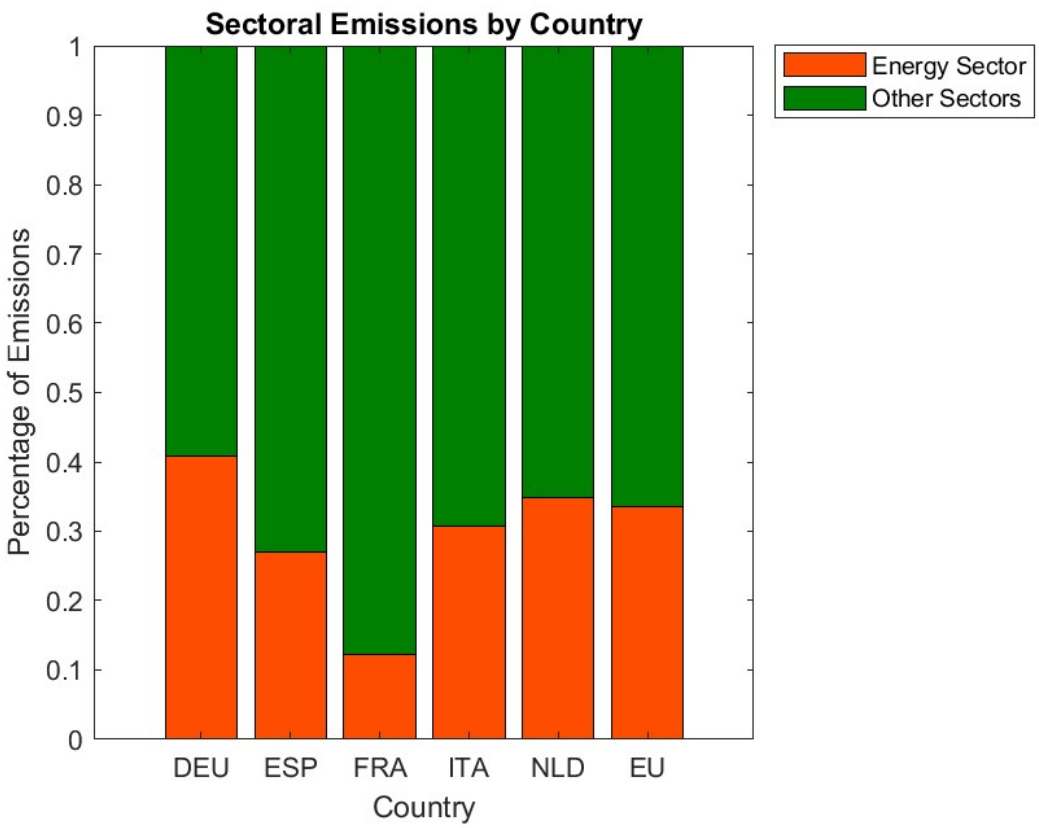
Figure 4: Emission Intensity in the EA



Notes: The figure depicts the emissions intensity in the energy sector and in the total industry for the top 5 EA economies, along with the EA mean over the estimated period (2013 – 2019).

As for the price of carbon, we proceed in two steps. We first find the value of the abatement function level θ_1 that is consistent with the observed mean EUA price of 7.54 euros. This also takes into account the split of emissions across sectors (see Figure 5) and the emission intensity of the energy sector. Then, we assume that the implied level of the EUA price was optimal over the 2013-2019 period and retrieve the value of Θ^T. More precisely, we find the value of Θ^T that equates the steady state level of the welfare in the model to the level of the welfare in the counterfactual optimal case. As we will show later, this does not imply that the economy in the estimated model behaves optimally. In particular, high volatility in the EUA price will generate losses in consumption for risk-averse agents that are more severe in the estimated case than in the optimal case.

Figure 5: Sectoral Emissions in the EA



Notes: The figure depicts the emissions split between the energy sector and the rest of the industry for the top 5 EA economies, along with the EA mean over the estimated period (2013 – 2019).

Finally, we use the decay rate of emissions η to ensure that the stock of emissions in the atmosphere is consistent with the mean level of emissions observed during the studied period and we set the public consumption to GDP ratio $\bar{g}/\bar{y}$ at 0.22.

The comprehensive list of calibrated parameters, along with the targeted economic and environmental moments they allow us to replicate, can be found in Table 3 and Table 4, respectively.

4.4 Estimation

Our model's shock processes and trends are estimated using the Kalman Filter. We employ the Metropolis-Hastings algorithm to approximate the posterior distribution, constructing our results based on four distinct chains. The estimation outcomes are concisely presented in Table 5, where we display both the prior and posterior densities of the estimated parameters.

The robust identification of the majority of these parameters indicates the informativeness of the data used. Despite some constraints in pinpointing their exact values, the trends in emissions and output are clearly discernible. Notably, our model's estimation is able to capture the decoupling between output and emissions. This is evidenced by the negative value of γ^x and the positive value of γ^y, a pattern that persists even with the application of normal priors centered around zero for both trends.

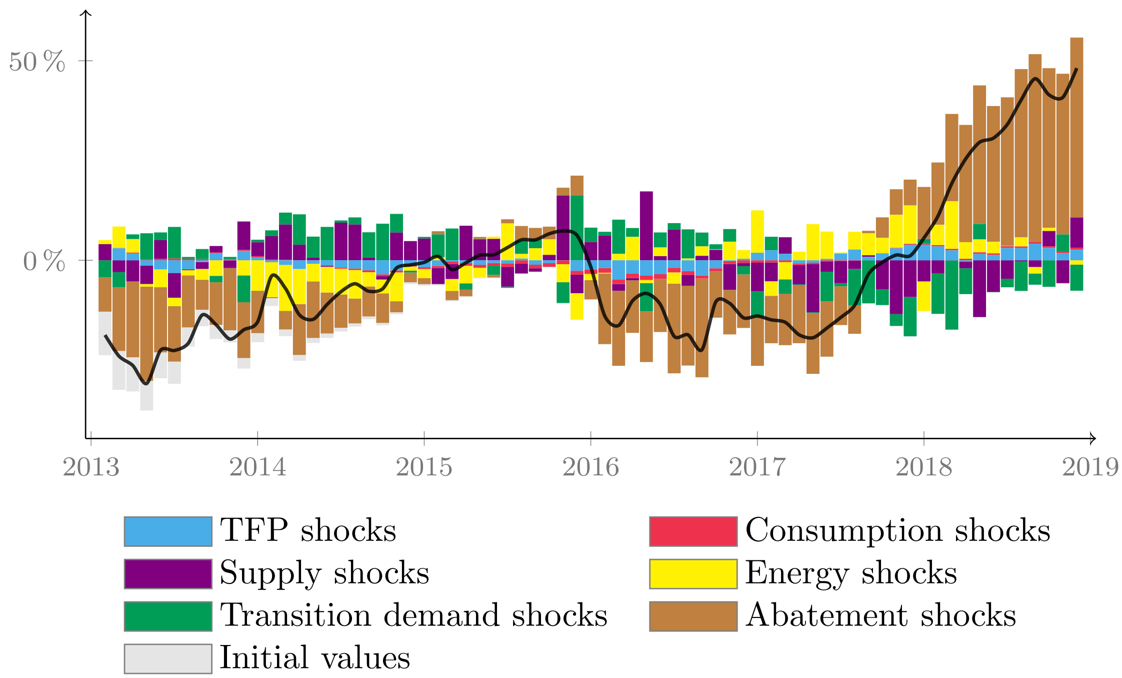
5 The Drivers of EU Carbon Permits and Optimal Carbon Price

In this section, we turn to one of the primary research questions: identifying the proportion of the carbon price in the EU ETS that is driven by supply factors and five distinct demand factors. To do so, we utilize the parameters and shock series previously estimated in our model. To ensure the robustness of our findings, we cross-reference these results with real-world data (EU total expenditure in green technologies) to verify their accuracy and assess the correlation between our model's outputs and observed data. Following this analysis, we undertake a comparative study. We compare the carbon price observed in the EU ETS with the case where the environmental regulator sets the carbon price optimally –essentially equating it to the social cost of carbon (SCC) as determined by our estimated parameters and shock series. This comparison allows us to evaluate how the actual carbon market in the EU ETS deviates from the theoretical optimum, and measure the additional volatility present in the EU ETS market compared to the SCC over the studied period.

5.1 Uncovering the Drivers in the EU ETS Carbon Market

Figure 6 presents the historical decomposition of the changes in EUA futures prices from 2013 to 2019. The black line represents the percentage change in the de-trended EUA futures price over this period, providing a clear view of the overall price movement. The colored bars indicate the relative contribution of each factor influencing the carbon permit price. These factors include both supply and various demand-side elements, each contributing differently to the observed price changes.[19]

Figure 6: Historical decomposition of changes in EUA futures price



Notes: The figure depicts the path of the EUA futures price (black line) broken down into different drivers over the estimated period from 2013 to 2019).

The figure indicates that abatement, energy, transition demand, and supply are the primary drivers of changes in carbon permit prices. In contrast, TFP and consumption have a limited impact on carbon prices. This outcome aligns with expectations, as TFP and consumption primarily affect energy firms' production indirectly, rather than directly influencing their emissions, which are the main drivers of permit demand.[20]

As discussed in Section 2, theoretical models suggest that the demand for permits and the cost of permits are fundamentally driven by abatement. This is precisely what we observe in

[19] Figure 11 in Appendix C presents the same decomposition, with shocks grouped into demand and supply categories, offering an alternative perspective on the factors driving price changes.

[20] TFP shocks, for example, represent variations in productivity among final goods firms, such as the adoption of a new manufacturing technique that boosts output. Similarly, consumption shocks reflect changes in consumer demand patterns, like a decrease in the subjective discount factor, leading consumers to defer consumption. While these factors can directly impact interest rates and firm profits, they only indirectly influence the demand for emission permits.

Figure 6, where abatement emerges as the most significant driver, particularly during the last period of Phase 3 when the Market Stability Reserve (MSR) was being implemented. The MSR, a rules-based mechanism designed to adjust the cap by removing supply, effectively increased scarcity in the market.[21] As a result, abatement became even more critical as a strategy for firms to meet their cap targets.

The other three factors –energy, transition demand, and permit supply– are more or less equally relevant in driving permit prices. The importance of energy aligns with empirical studies that highlight the strong connection between the EU ETS market and energy markets, emphasizing the influence of energy shocks. Between 2013 and 2019, the European Commission (EC) and several Member States (MBs) implemented and amended several key climate and energy policies aimed at improving energy efficiency, promoting renewable energy generation, and phasing out fossil fuels.[22] The relevance of uncertainty in transition demand is evident throughout this period at varying levels, reflecting the impact of these policy changes on permit demand. Figure 6 also confirms that regulatory uncertainty has been a crucial driver of permit prices, consistent with the findings from the empirical literature discussed earlier. The conversation about structural reform of the EU ETS began in earnest around 2013, when it became increasingly clear that the EU ETS was facing significant challenges, including an oversupply of permits that led to a collapse in carbon prices in 2013. In 2014, the EC proposed the creation of a supply-adjustment mechanism, the MSR, as a long-term solution to address the structural imbalance between supply and demand in the carbon market. The MSR was designed to automatically adjust the supply of allowances to ensure demand-supply stability. Parliamentary discussions continued throughout 2017, leading to the formal adoption of revisions to the EU ETS for its fourth trading phase (2021-2030). These revisions included a strengthening of the MSR and other measures aimed at tightening the cap on emissions. The relevance of the supply factor is particularly visible during the key periods of 2014 and 2017, reflecting the significant impact of these policy developments on permit demand and permit prices.

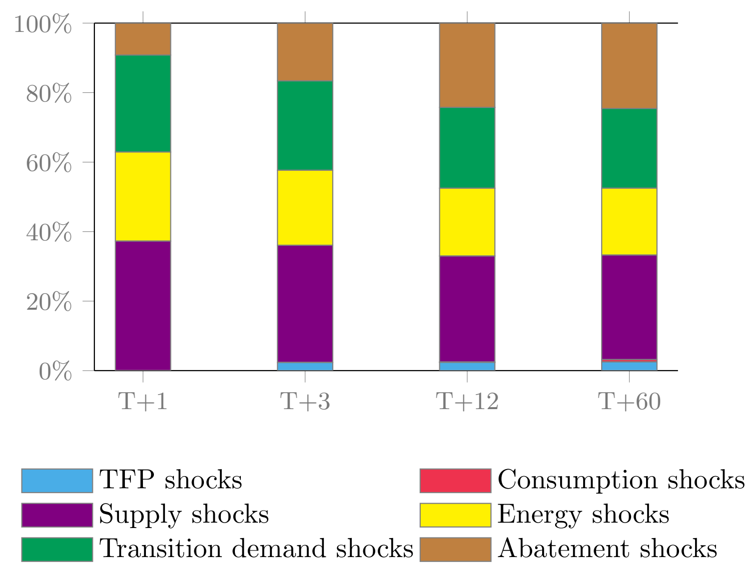
Figure 7 illustrates the contribution of each driver to the variance of the carbon price

[21] Being a rules-based mechanism, the MSR does not manifest as a traditional supply shock.

[22] While the EC revised the Energy Efficiency and Renewable Energy Directives to set more ambitious 2030 targets, several MBs have made policy reversals or delays, including Germany postponing its coal phase-out and reducing renewable subsidies, the UK cutting solar subsidies and delaying fossil fuel phase-outs, Poland continuing its coal reliance, France slowing its nuclear phase-out while supporting gas, and Italy delaying renewable energy auctions. Such actions can create significant shocks in the transition to a low-carbon generation.

across various time horizons (1, 3, 12, and 60 months). The four primary factors identified earlier –abatement, energy, transition demand, and supply– account for virtually the entire variance in carbon prices across all these time horizons.

Figure 7: EUA Futures Price Variance Decomposition



Notes: The figure displays the variance decomposition of the EUA futures price based on different horizons: one month, three months, one year, and five years. This represents the theoretical variance decomposition of the permit price, taking into account the estimated variances of shocks.

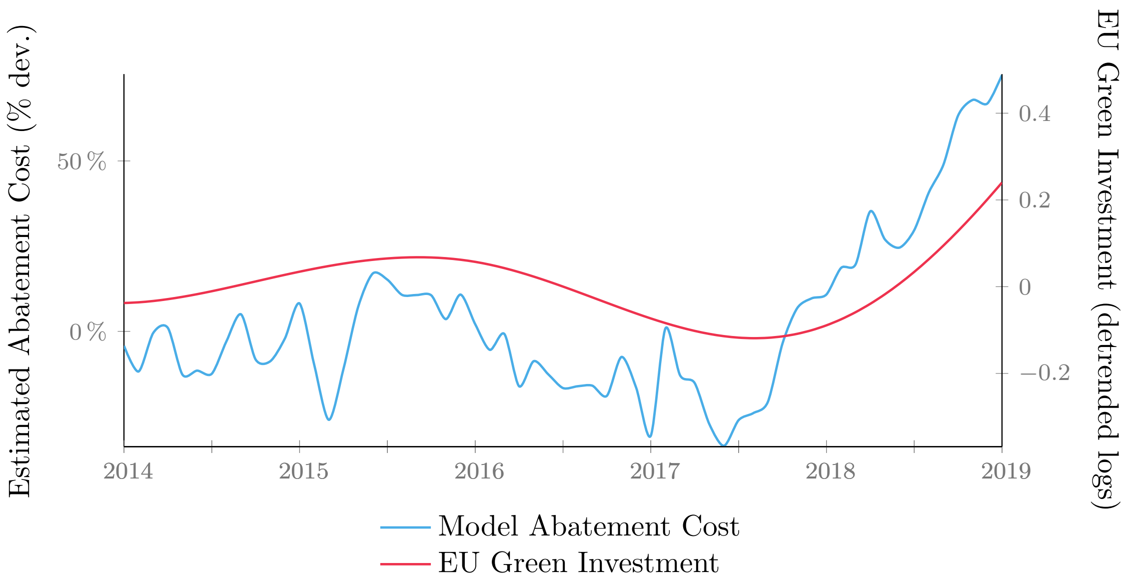
In the short term, all four factors play significant roles in driving price fluctuations. However, as we move to longer horizons, the relative importance of energy and supply shocks begins to diminish. In contrast, abatement shocks become increasingly significant. This shift underscores the enduring impact of investments in abatement technologies. Although initially costly, these investments lead to long-term improvements in firms' emission efficiencies, which increasingly dominate the variance in carbon prices over extended periods.

5.2 How does our estimated series compare to the actual data?

To examine the validity of our interpretation of the abatement factor–the primary driver influencing carbon permit prices–we cross-referenced it with closely related real-world data to ascertain its accuracy and correlation.

As such, we compare our derived estimated series for abatement against the annual data reflecting the EU's net-zero emission total expenditure. Figure 8 presents both the interpolated EU data on total climate mitigation expenditure and our model's estimated abatement investment. Both series exhibit similar trends and business cycle fluctuations, indicating a strong connection between our model's abatement estimates and real-world green investment activities. To facilitate a detailed comparison, we transformed the annual data into a monthly format using Cubic Spline Interpolation, aligning it with our monthly abatement estimation series for greater accuracy.

Figure 8: Estimated Abatement Costs and Climate Mitigation Investment Data



Notes: The figure displays the estimated abatement costs as a deviation of their steady state, alongside the actual data on climate mitigation investment for the EU in detrended log million euros.

It's important to note that the data we use primarily reflects the EU's overall commitment to green investments rather than the explicit abatement costs featured in our model.[23] Nevertheless, this comparison offers valuable insight into whether our model's estimations align with actual green investment expenditures.

5.3 EU ETS cap-and-trade and optimal carbon price

Market participants have frequently raised concerns that carbon prices in the EU ETS are too volatile, especially given their role in providing signals to incentivize structural, long-term projects – a sentiment recently highlighted in the Financial Times.[24] In the previous

[23]This data measures the total amount spent from the annual budgets of EU Member States and the European Investment Bank to contribute to the USD US$100 billion commitment for climate finance under the United Nations Framework Convention on Climate Change (source: DG CLIMA, EIONET).

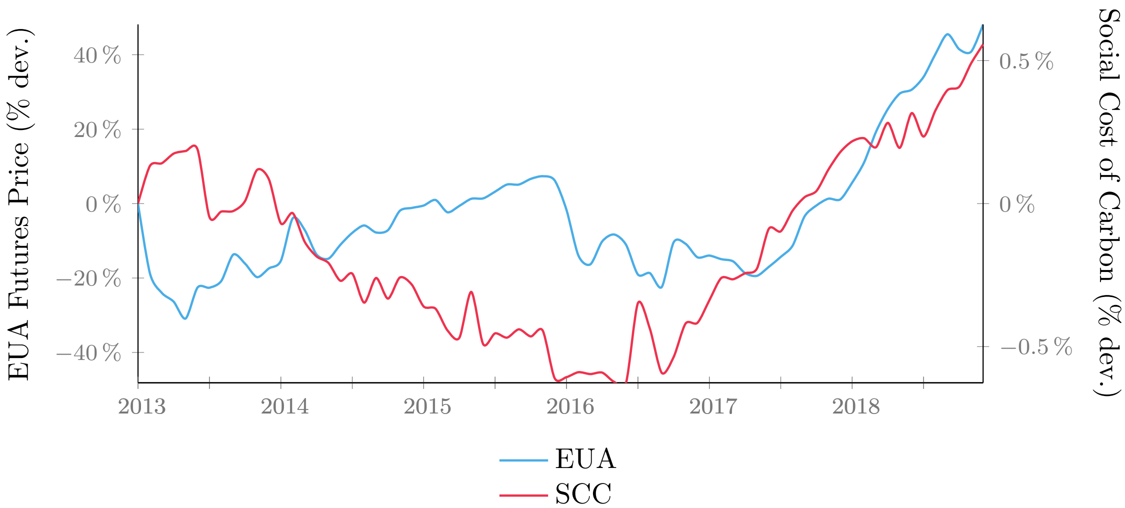
[24]'Europe's carbon price crash looks like serious market myopia', Financial Times February 28th 2024.

section, we identified the factors that most contribute to carbon price variability. In this section, to gauge the extent of this *extra* volatility in the EU ETS market, it is essential to establish a baseline for comparison.

To create this baseline, we construct a counterfactual scenario in which the environmental regulator aligns the carbon price with the estimated SCC. This approach would represent the first-best –the optimal carbon price– where the cap is optimally set to reflect the true cost of emissions, ensuring that the market efficiently allocates emissions in a decentralized economy. To compute the SCC, we use the estimated parameters and shock series, while replacing our cap and carbon price equation with the SCC.[25] In this scenario, policy uncertainty (supply shock) is non-existent. This means the carbon price is set optimally with full commitment from the regulator, with no political interference or subsequent alterations; once a policy is set, it remains unchanged. As a result, supply shocks do not exist under an SCC framework, as the optimal carbon price eliminates the need for adjustments based on regulatory uncertainty. However, demand shocks such as those related to abatement, transition demand, and energy continue to persist. The SCC is designed to account for these shocks, reflecting changes in the cost of emissions under varying economic conditions. With the SCC, we can then quantify the additional carbon price volatility present in the EU ETS compared to a system where the SCC is implemented. Later, this allows us to measure the welfare loss that arises from demand and supply uncertainty in the current EU ETS setup.

[25] The SCC is formally derived in the Appendix F.

Figure 9: EUA Carbon Price vs SCC Variations



Notes: The figure shows the deviations of the estimated EUA futures price and the SCC in percentage deviations from their respective steady states.

Figure 9 displays percentage deviations from the steady-state for both the estimated EUA futures price and the estimated SCC. While the trajectories of the EU ETS carbon price and the SCC generally mirror each other, the SCC's fluctuations are significantly smaller –about eighty times less than those of the EU ETS carbon price. This stark difference underscores the considerable additional volatility present in the EU ETS market compared to the SCC. Table 1 compares key statistical moments between the estimated EU ETS cap policy and the counterfactual SCC. By continuously equalising the marginal costs of emissions with the marginal benefits of emission reduction, the SCC stabilizes abatement costs, resulting in a carbon price with virtually zero volatility. This remarkable stability in both the carbon price and the abatement cost is achieved at the expense of slightly greater variations in emissions at the business cycle frequency.

Setting and implementing the SCC is complex. A more practical approach could involve setting the cap according to a specific emission trajectory and then adjusting the number of permits allocated in response to relevant shocks. In the subsequent section, we explore the potential of such adaptive carbon cap rules, where the allocation of permits is conditioned on the evolution of the most relevant carbon price drivers.

5.4 The cost of carbon price business cycle fluctuations

In this section, we explore the costs associated with business cycle fluctuations in consumption equivalence (CE) that arise due to uncertainty and price volatility linked to the SCC or the carbon permit price in the EU ETS. These fluctuations in carbon prices can introduce significant variability in economic outcomes, leading to welfare losses for risk-averse agents who prefer stable and predictable consumption paths (Lucas (1987)).

To quantify these costs, we compare the welfare in a deterministic environment (where all shocks are absent) with that in a stochastic environment (where shocks to carbon prices and other variables are present). Specifically, we proceed as follows. First, we compute the lifetime utility given by the value function of our representative household in the deterministic case. This serves as a baseline for understanding welfare under stable economic conditions. Next, we calculate the lifetime utility in the stochastic framework, where the economy is subject to various shocks, including those affecting carbon prices. To quantify the welfare difference, we introduce a parameter, Δ, that represents the necessary adjustment in consumption equivalence to equalize welfare between the deterministic and the two stochastic cases. We denote the deterministic welfare function by Welfare_t^D and the stochastic counterpart by Welfare_t^S.

$$\text{Welfare}_t^D = u(C_t, D_u(T_t^o)) + \beta \mathbb{E}_t\{\text{Welfare}_{t+1}\}$$

$$\text{Welfare}_t^S = u((1-\Delta)C_t, D_u(T_t^o)) + \beta \mathbb{E}_t\{\text{Welfare}_{t+1}\}$$

Since risk-averse agents dislike fluctuations, the welfare in the stochastic economy is lower than in the deterministic scenario. Mathematically, this is represented by the inequality $\mathbb{E}(\text{Welfare}_t^S) < \text{Welfare}_t^D$, where $\mathbb{E}(\text{Welfare}_t^S)$ is the expected welfare in the stochastic environment. We then determine the value of Δ –the compensation needed in terms of CE– to bridge the gap between welfare in the stochastic and deterministic cases, or $\mathbb{E}(\text{Welfare}_t^S) = \text{Welfare}_t^D$. This compensation represents the additional consumption required to make agents indifferent between the fluctuating and stable environments. In this particular exercise, we exclude shocks related to general consumption and the TFP of final firms to isolate the impact of climate and energy-related shocks. The value obtained for $|\Delta|$ is $1.26e^{-5}$ when the carbon price is the SCC, and $1.74e^{-5}$ when the carbon price is the one observed in the EU ETS. These values correspond to a yearly cost of business cycle fluctuations that amounts to approximately 0.006 percent in consumption equivalence (CE) terms, comparing the first-best policy scenario with the EU ETS carbon cap case. While 0.006 percent might seem like a very small number, this estimate aligns with the seminal work of Robert Lucas, who in 1987 calculated that the welfare cost of business cycles was around 0.008 percent of consumption.

6 Responsiveness of Carbon Cap Rules

In the preceding section, we highlighted the pronounced volatility in the permit price observed during phase 3 of the EU ETS market. This volatility is not merely a statistical observation but carries significant real-world implications. It is important to understand that while some degree of volatility is anticipated in any cap-and-trade system, the levels observed in the EU ETS market during this phase were particularly high. Such volatility can be a double-edged sword: it can indicate a market's responsiveness to changing conditions, but it also introduces unpredictability that undermines the objective of providing consistent price signals necessary to incentivize long-term abatement projects.

Thus, for policymakers this volatility presents challenges as it can undermine the very goals the cap-and-trade system is designed to achieve. If prices are too volatile, firms might hesitate to invest in long-term emission reduction strategies, fearing that the costs might outweigh the benefits if prices swing too widely (Martin, Muuls, and Wagner (2011), and European Parliament (2022), among others). Market participants, especially firms, bear the brunt of this volatility. Excessive price fluctuations introduce a level of market uncertainty that can be challenging to navigate. Such unpredictability makes it difficult for firms and investors to commit to substantial long-term investments, especially when these investments are potentially irreversible (Calel (2020) and Taschini (2021)). The fear of making a costly mistake due to volatile prices can deter companies from investing in capital-intensive projects or adopting new technologies. This hesitancy can slow down innovation and progress, particularly in sectors where upfront investments are crucial for future decarbonization. Moreover, the inability to accurately forecast returns on these investments due to price volatility can lead to missed opportunities and hinder strategic planning. Furthermore, this volatility can spill over into financial markets (Benmir and Roman (2020)). Unpredictable carbon prices increase risks for firms, leading to higher risk premia and, consequently, higher borrowing costs as lenders seek greater returns to offset the risk. Over time, this can hinder firms' investment decisions, potentially slowing the transition to greener technologies.

A contingency mechanism, where the cap adjusts dynamically in response to changes in emissions and key factors like abatement, could help reduce permit price volatility (Newell and Pizer (2008), Doda (2016b), Kollenberg and Taschini (2016), and Karp and Traeger (2023)). By aligning the supply of permits more closely with the evolving demand, such a mechanism would provide a more stable and predictable pricing environment, ensuring that the market remains responsive while minimizing the unpredictability that hinders long-term abatement investments. Implementing a conditional supply of permit allowances that allows for a dynamic per-period cap could effectively manage price volatility in the allowance market while keeping emissions on the desired trajectory. To achieve this, we introduce a Carbon Cap Rule (CCR) designed to address the uncertainties primarily associated with two critical driving factors: abatement, transition demand, energy, and regulatory uncertainty. The CCR adjusts the quantity of emission permits (Q_t) in the market. This adjustment is based on deviations from the de-trended steady-state emissions ($\bar{e}^E$) and abatement costs ($\bar{z}$):

$$\mathrm{Q}_t = \overline{\mathrm{Q}} + \phi_e \frac{(e_t^E - \bar{e}^E)}{\bar{e}^E} + \phi_z \frac{(z_t - \bar{z})}{\bar{z}},$$

where ϕ_e and ϕ_z are coefficients that determine the sensitivity of the cap to changes in emissions and abatement costs, respectively. This approach ensures that the cap is not static but dynamically adjusts in response to abatement costs and emissions variations. The CCR functions similarly to the Taylor rule in monetary policy, offering a structured approach to adjusting emission caps in response to abatement (thus abatement costs) and regulatory shocks (and subsequently energy supply-as energy is a byproduct of emissions). Just as the Taylor rule guides central banks in adjusting interest rates based on economic indicators like inflation and output gaps, the CCR provides a formulaic method for dynamically managing emission caps in response to key environmental and economic factors.[26]

We now examine how the shock sensitivity of the CCR differs from that of the SCC curve. To do this, we use the parameters and shock series previously estimated, substituting the carbon price equation in our model with the CCR formula.[27] Next, we identify the

[26]The concept of the proposed CCR draws also parallels with the "target-consistent pricing" approach, a notion championed by among others Stern, Stiglitz, Karlsson, and Taylor (2022). This method pivots around the idea that the Social Cost of Carbon should be formulated in a manner that inherently aligns with the objectives set out in the Paris Agreement. Instead of determining the SCC based on estimated damages from an additional ton of carbon dioxide, this approach works backward: it starts with the goals of the Paris Agreement and then calculates the SCC required to achieve those specific targets. This perspective ensures that pricing is consistent with broader climate objectives and should provide a clear policy and, crucially, price signal for the necessary transition.

[27]Note that we retain the supply shock, even though one could contend that if a carbon cap rule were in place, supply might not influence the emissions path. Thus, our counterfactual represents a conservative

optimal values for the coefficients ϕ_e and ϕ_z by finding the values that minimize the standard deviation of the carbon price, thereby reducing volatility. To refine our initial guesses for these coefficients, we employ a quasi-Newton method. The economy's path is simulated to the second order for each parameter pair in the CCR until the algorithm reaches convergence, ensuring that the solution is robust. Our results show that ϕ_z is positive, as expected, meaning that in response to a positive shock to abatement costs, the regulator following the CCR would increase the cap to ease abatement cost pressures and reduce deviations from abatement goals. Conversely, ϕ_e is optimally negative, suggesting that when firms exceed emission targets, the regulator would tighten emission constraints.

	ETS Cap Policy Estimated Column (1)	Social Cost of Carbon Optimal Column (2)	Carbon Cap Rule $\phi_z = 0.1853$ and $\phi_e = -0.0027$ Column (3)
Consumption (Std. Dev.)	1.74 %	1.78 %	1.73 %
Output - Industrial Prod (Std. Dev.)	1.11 %	1.11 %	1.11 %
Emissions (Std. Dev.)	0.9 %	2.44 %	2.46 %
Abatement Cost (Std. Dev.)	18.33 %	9.33 %	8.29 %
Carbon Price (in euros)	7.52	7.52	7.44
Carbon Price (Std. Dev.)	19.17 %	0.31 %	3.51 %

Table 1: Policy Scenarios Estimated Second Moments

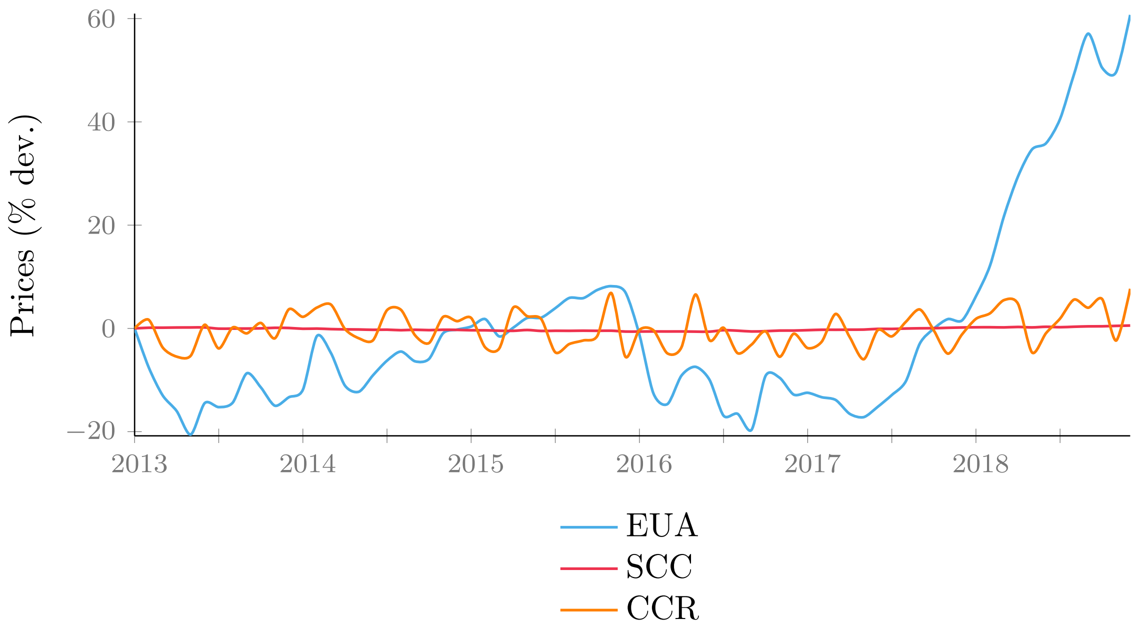
Notes: The table reports various moments under three cap policy scenarios. The first column corresponds to the estimated ETS model, the second column corresponds to the Social Cost of Carbon – the optimal case, and the third column corresponds to the Carbon Cap Rule (CCR). The CCR is $\mathrm{Q}_t = \overline{\mathrm{Q}} + \phi_e \frac{(e_t^E - \bar{e}^E)}{\bar{e}^E} + \phi_z \frac{(z_t - \bar{z})}{\bar{z}}$.

Table 1 presents key statistical moments across three different cap policy scenarios. The first column displays results from the estimated EU ETS model, the second represents the SCC (considered the optimal carbon pricing case), and the third details outcomes under the CCR. First, regarding macro aggregates, the inclusion of consumption habits allows for a better match between the standard deviation of consumption and output. Second, our proposed CCR reduces price volatility by about 55%. The CCR exhibits a significantly stronger response to deviations in steady-state abatement costs compared to deviations in steady-state emissions. This suggests that the rule prioritizes managing abatement costs over strictly adhering to per-period emission levels. This approach underscores the importance of keeping abatement at a manageable level, rather than strictly enforcing a set limit on emissions each year.

scenario where some unexplained volatility in emissions persists.

Considering the CCR's emphasis on managing abatement costs, it is unsurprising that the volatility of these costs under the CCR is similar to that in the SCC scenario. The volatility of emissions under the CCR is also very close to what it is under the SCC. In terms of carbon price volatility, the standard deviation under the CCR is higher than that under the SCC, but significantly lower than the volatility observed in the current ET ETS model. This suggests that although the CCR does not entirely eliminate price volatility, it significantly reduces the extremes seen in the existing EU ETS carbon market, thereby mitigating the unpredictability associated with abatement costs for firms. This effect is observable in Figure 10.

Figure 10: EUA vs SCC vs CCR Variations



Notes: The figure shows the deviations of the estimated EUA futures price, the counterfactual SCC, and the counterfactual CCR in percentage deviations from their respective steady states.

Crucially, the proposed CCR significantly reduces the business cycle costs associated with price volatility. As shown in Table 2, it cuts welfare costs in half, lowering them to approximately 0.0036 percent in terms of welfare losses, compared to the 0.006 percent welfare losses in CE observed under the EU ETS cap relative to the SCC. Reductions in welfare losses could be even more substantial if carbon prices were higher. During the estimation period, the EU ETS price remained relatively low, which helped to keep the associated welfare losses at a modest level. However, as carbon prices increase –reflecting a higher social cost of carbon and therefore more stringent environmental policies– the economic impact of price volatility and uncertainty could become more pronounced.

	ETS Cap Policy $\lvert\Delta\rvert = 1.74e^{-5}$ Column (1)	Social Cost of Carbon $\lvert\Delta\rvert = 1.26e^{-5}$ Column (2)	Carbon Cap Rule $\lvert\Delta\rvert = 1.56e^{-5}$ Column (3)
Yearly welfare loss (in %CE) w.r.t SCC	0.006	—	0.0036

Table 2: Business Cycle Welfare Cost

Notes: The table reports welfare business cycle costs of uncertainty for the three cap policy scenarios studied. The first column corresponds to the estimated ETS model, the second column corresponds to the Social Cost of Carbon – the optimal case, and the third column corresponds to the Carbon Cap Rule (CCR). The CCR is $\mathrm{Q}_t = \overline{\mathrm{Q}} + \phi_e \frac{(e^E_{t-1} - \bar{e}^E)}{\bar{e}^E} + \phi_z \frac{(z_t - \bar{z})}{\bar{z}}$.

The CCR has the potential to serve as a foundational rule for a Central Carbon Bank, an institution proposed to oversee carbon market dynamics (de Perthuis (2011), Pahle and Edenhofer (2021), Blanchard and Tirole (2021)). This regulatory authority would be responsible for managing the supply of permits, with the ability to intervene as necessary to stabilize permit prices. While our study does not explore the specific governance structure of a Central Carbon Bank, we acknowledge that the CCR could effectively guide its primary function of managing the cap.

7 Conclusion

Cap-and-trade systems are the primary market-based approach for regulating greenhouse gas emissions, but recent years have exposed significant shortcomings, particularly in terms of high permit price volatility.

Analyzing EU ETS data from 2013 to 2019 using a two-sector DSGE model, we identify the primary drivers of carbon permit prices: abatement, energy prices, transition demand, and regulatory supply. We introduce an innovative method to infer abatement cost shocks by examining deviations from expected emissions trends linked to regulatory changes and combining this analysis with permit price data, as the permit price should reflect the marginal cost of emissions reduction. The volatility in the EU ETS market is approximately eighty times greater than that observed under a hypothetical scenario where the carbon price is aligned with the Social Cost of Carbon. This excess volatility leads to welfare losses, quantified at approximately 0.006 percent in consumption equivalence terms, highlighting the economic cost of the current permit price volatility.

To address this issue, we propose a Carbon Cap Rule that dynamically adjusts the emission cap in response to deviations from steady-state emissions and abatement costs. By applying the CCR, we obtain a significant reduction in price volatility—by about 55% compared to the volatility under the current EU ETS cap. CCR not only stabilizes carbon prices but also reduces welfare losses, making it a viable alternative to the current cap-and-trade framework. The CCR could serve as a foundational rule for a proposed Central Carbon Bank, an institution designed to oversee and manage carbon market dynamics. Similar to how the Taylor rule provides a structured method for central banks to adjust interest rates, the CCR offers a formulaic approach to managing emission caps, ensuring that carbon pricing remains stable and predictable. By aligning the cap more closely with market fundamentals, the CCR enhances the efficiency of the carbon market, supporting long-term investments in emissions reduction and contributing to a more effective transition to a low-carbon economy.

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Appendix A Data

We assembled our dataset by integrating multiple sources. This includes macroeconomic goods productivity data and consumption patterns obtained from National Statistical Offices and Eurostat; energy production and pricing information sourced from Bloomberg; carbon dioxide emissions data from the Emissions Database for Global Atmospheric Research (EDGAR); and data on European Union Allowance (EUA) futures prices from the Intercontinental Exchange (ICE).[28] We restrict our empirical study to countries within the European Union Emission Trading System (EU ETS) framework,[29] and the period January 2013 to December 2019. This period aligns with phase 3 of the EU ETS and includes the UK, which remained a part of the European carbon market until 2020.

Goods productivity and consumption patterns From Eurostat, we have compiled data on the consumption preference index for each country to capture the evolving trends in consumer behavior and preferences. Additionally, we have gathered data on the industrial production index for each European state, providing a measure of the extent of industrial activity.

Energy supply data From Bloomberg, we compile data on energy production, focusing on both the volume of energy produced and the corresponding price levels. In line with the empirical literature discussed earlier, we consider three critical energy sources: Brent crude oil, natural gas, and coal. This data collection enables us to closely monitor the supply of energy, an essential determinant of the price of emission allowances.

Carbon dioxide emissions The Emissions Database for Global Atmospheric Research (EDGAR) provides estimates for emissions of the three main greenhouse gases (CO_2, CH_4, N_2O) per sector and country. This comprehensive dataset enables us to study emission dynamics at a high frequency.

[28] EUAs grant permission to emit one tonne of carbon dioxide (CO_2) or an equivalent amount of another greenhouse gas. "EUA Futures" refers to futures contracts based on these allowances.

[29] The EU ETS currently operates in 30 countries: the 27 EU member states plus Iceland, Liechtenstein, and Norway. The United Kingdom left the EU on 31 January 2020 but remained subject to EU rules until 31 December 2020. In our analysis, we consider the 27 EU member states and the United Kingdom. Due to data constraints, we omit Norway and Liechtenstein.

Emission allowance prices From the Intercontinental Exchange (ICE), we retrieve data on daily carbon futures contracts, the EUA futures contracts. Our data collection includes the daily prices of these EUA futures contracts, which we then convert from a daily to a monthly frequency. By examining EUA prices, we gain valuable insights into the market's response to innovations in abatement technologies, a less well-observed driver of the EU ETS.

Climate sentiment transition index We use Bua et al. (2022) climate sentiment index capturing transition risk to evaluate the wiliness of energy firms to abate. The authors use Reuters news feeds to construct the index for the EU.

Appendix B Model Calibration and Estimation

Table 3: Parameters Value

Parameter	Value	Definition
σ^U	1.5	Risk Aversion
β	0.9986	Discount Factor
α^E	0.33	Elasticity to Capital Input in Energy Production
α^{NE}	0.33	Elasticity to Capital Input in Non-Energy Production
χ	0.02	Share of Energy in the CES
σ	0.20	Substitution Parameter in the CES
δ	0.0083	Depreciation of Energy and Non-Energy Capital
φ^E	0.0055	Emission Intensity in Energy Production
φ^{NE}	0.0002	Emission Intensity in Non-Energy Production
Θ^T	26.29	Dis-utility Sensitivity to Temperature
η	0.0004	Decay Rate of Emissions in the Atmosphere
ζ_1^o	0.50	Climate Transient Parameter
ζ_2^o	0.00125	Climate Transient Parameter
θ_1	0.239	Level of the Abatement Cost Function
θ_2	2.7	Curvature of the Abatement Cost Function
$\frac{\bar{g}}{\bar{y}}$	0.22	Government Spending to Output Ratio

Table 4: Moments matching

Variable	Label	Target	Source
ETS Mean Carbon Price (euros)	τ	7.54	ICE
Cumulative Emission (World, GtC)	X	800	Copernicus (EC)
Monthly Emission Flow (World, GtCO2)	$E^T + E^*$	4.51	Ourworldindata
Share of EU27 in World Emissions (%)	$E^T/(E^T + E^*)$	6.73	Ourworldindata
Share of Emissions from Energy Generation in the EU (%)	E^E/E^T	33.56	OECD
Emission intensity in the EU (kCO2 / euros)	E^T/Y	0.20	OECD
Emission intensity from Energy Generation in the EU (kCO2 / euros)	E^E/Y	0.07	OECD
Abatement level (percentage of energy emissions)	μ	0.20	EDGAR (EC)
Temperature	T^o	1.00	NOAA

Notes: All the values reported in this table are perfectly matched by the model at the steady state.

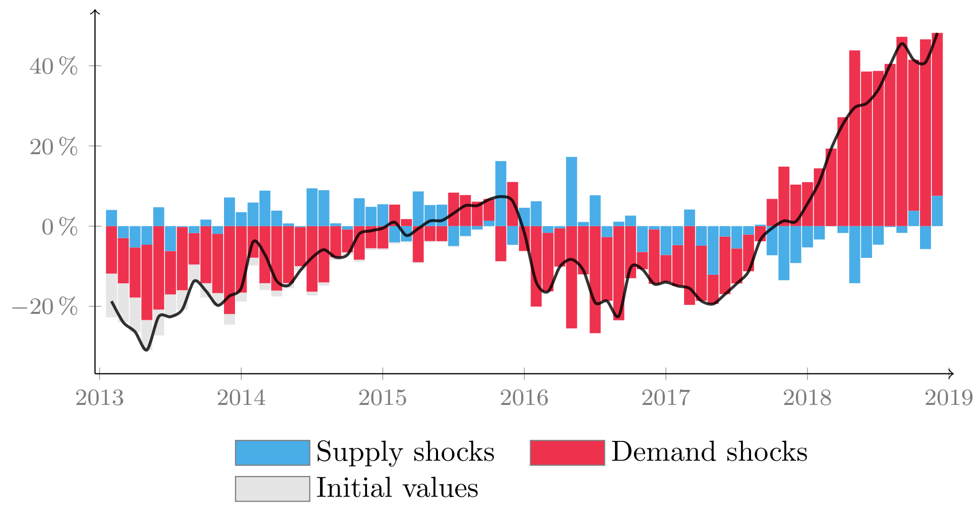
Table 5: Estimated Parameters

		Prior Distributions			Posterior Distributions	
		Distribution	Mean	Std. Dev.	Mean	[0.05 ; 0.95]
Shock processes:						
Std. Dev. Goods Productivity	σ_A	$\mathcal{IG}_2$	0.10	0.05	0.02	[0.01 ; 0.02]
Std. Dev. Energy Productivity	σ_{A_n}	$\mathcal{IG}_2$	0.10	0.05	0.01	[0.01 ; 0.02]
Std. Dev. Energy Price	σ_p	$\mathcal{IG}_2$	0.10	0.05	0.09	[0.07 ; 0.11]
Std. Dev. Climate Sentiment	σ_{φ_E}	$\mathcal{IG}_2$	0.10	0.05	0.02	[0.01 ; 0.02]
Std. Dev. Consumption	σ_B	$\mathcal{IG}_2$	0.10	0.05	0.10	[0.09 ; 0.13]
Std. Dev. Abatement Cost	σ_Z	$\mathcal{IG}_2$	0.10	0.05	0.06	[0.05 ; 0.07]
Std. Dev. Allowances Supply	σ_S	$\mathcal{IG}_2$	0.10	0.05	0.02	[0.01 ; 0.02]
AR(1) Goods Productivity	ρ_A	$\mathcal{B}$	0.30	0.10	0.49	[0.32 ; 0.68]
AR(1) Energy Productivity	ρ_{A_n}	$\mathcal{B}$	0.30	0.10	0.35	[0.018 ; 0.54]
AR(1) Energy Price	ρ_p	$\mathcal{B}$	0.30	0.10	0.36	[0.22 ; 0.49]
AR(1) Climate Sentiment	ρ_{φ_E}	$\mathcal{B}$	0.30	0.10	0.34	[0.21 ; 0.50]
AR(1) Consumption	ρ_C	$\mathcal{B}$	0.30	0.10	0.21	[0.09 ; 0.30]
AR(1) Abatement Cost	ρ_Z	$\mathcal{B}$	0.30	0.10	0.86	[0.83 ; 0.89]
AR(1) Allowances Supply	ρ_S	$\mathcal{B}$	0.30	0.10	0.31	[0.15 ; 0.50]
Measurements errors:						
Consumption Survey		$\mathcal{U}$	0.0001	0.003	0.010	[0.009 ; 0.010]
Industrial Production		$\mathcal{U}$	0.0001	0.003	0.010	[0.009 ; 0.010]
Emissions		$\mathcal{U}$	0.0001	0.007	0.025	[0.024 ; 0.025]
Structural Parameters:						
TFP Trend	$(\gamma^y - 1) \times 100$	$\mathcal{U}$	0.00	0.29	0.17	[0.05 ; 0.27]
Emissions Trend	$(\gamma^x - 1) \times 100$	$\mathcal{U}$	0.00	0.29	-0.28	[-0.50 ; -0.07]

Notes: $\mathcal{IG}_2$ denotes the Inverse Gamma distribution (type 2), $\mathcal{B}$ the Beta distribution, and $\mathcal{N}$ the Gaussian distribution.

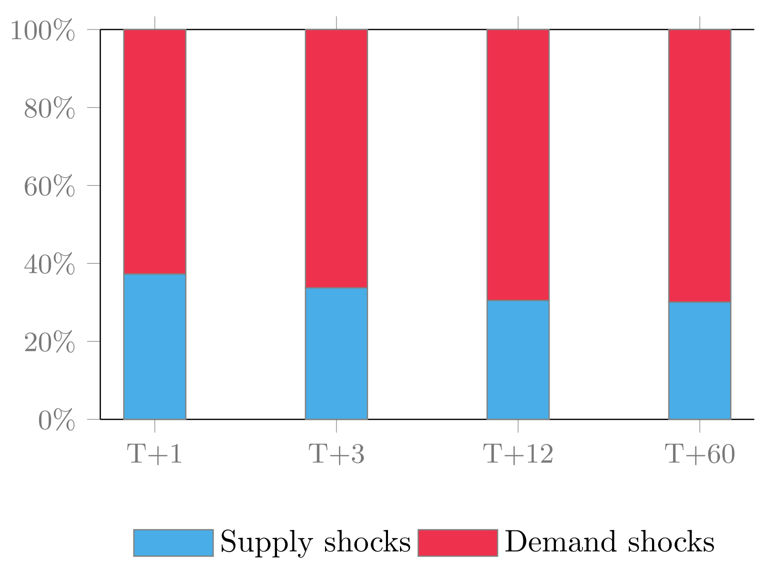
Appendix C Demand and Supply Shocks

Figure 11: EUA Futures Price Historical Decomposition



Notes: The figure depicts the path of the EUA futures price (black line) broken down into different drivers over the estimated period (2013 – 2019).

Figure 12: EUA Futures Price Variance Decomposition

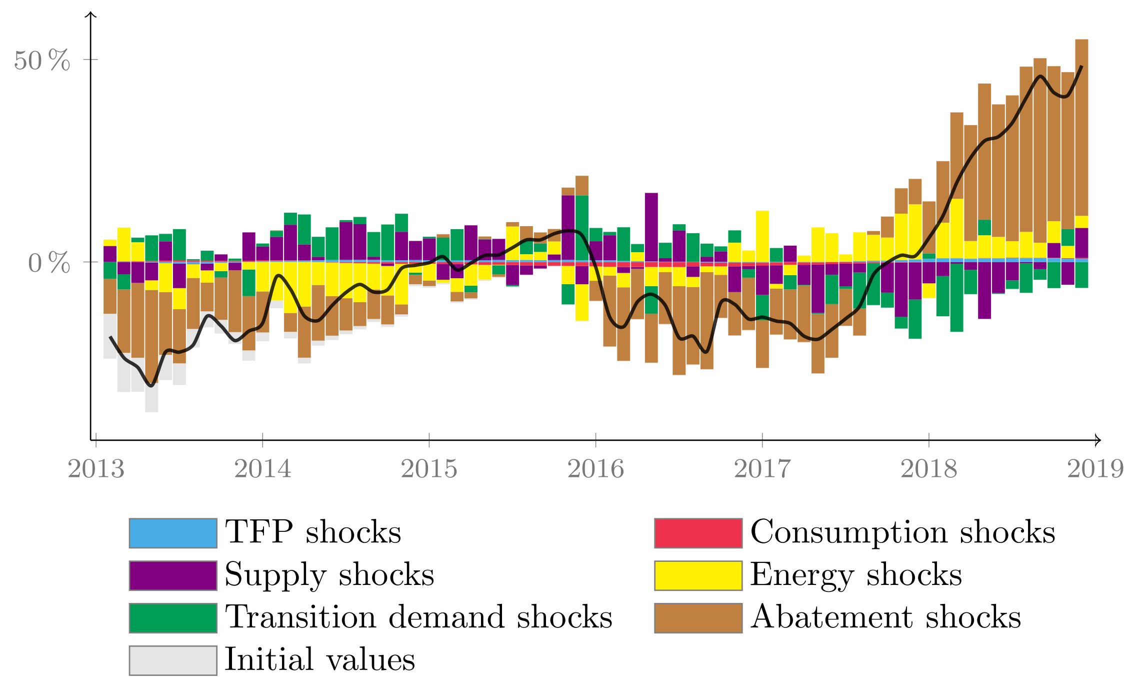


Notes: The figure displays the variance decomposition of the EUA futures price based on different horizons: one month, three months, one year, and five years. This represents the theoretical variance decomposition of the permit price, taking into account the estimated variances of shocks.

Appendix D Case of Cobb-Douglas Production Function

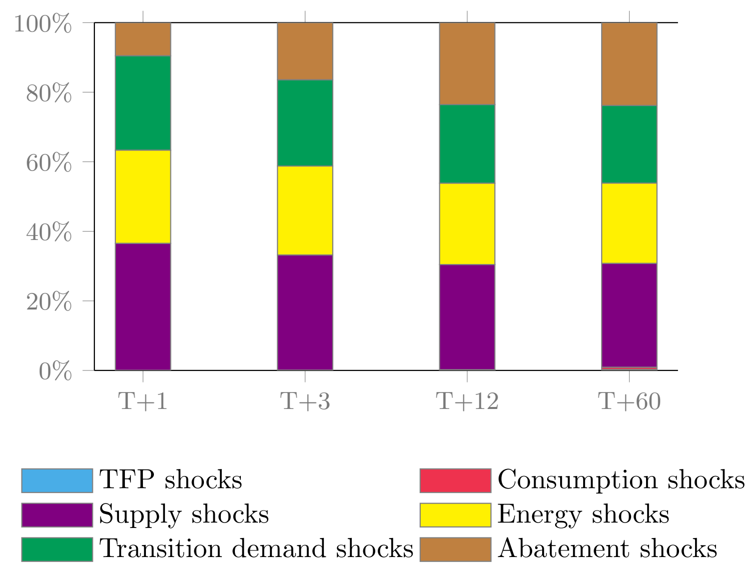
In this section we present the estimation results where production is modeled following a Cobb-Douglas function. We note that the results are sensitively similar to the CES case. The most noticeable difference is the increased role of energy shocks. Figure 13, figure 14, and figure 15 present the shock decomposition, variance decomposition, and the comparison between the SCC and EU ETS.

Figure 13: EUA Futures Price Historical Decomposition



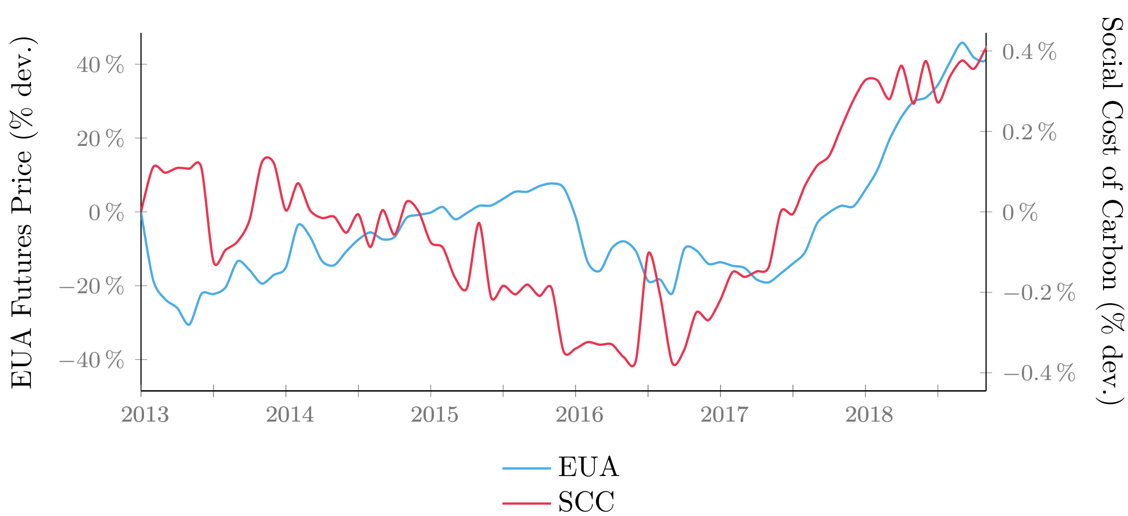
Notes: The figure depicts the path of the EUA futures price (black line) broken down into different drivers over the estimated period (2013 – 2019).

Figure 14: EUA Futures Price Variance Decomposition



Notes: The figure displays the variance decomposition of the EUA futures price based on different horizons: one month, three months, one year, and five years. This represents the theoretical variance decomposition of the permit price, taking into account the estimated variances of shocks.

Figure 15: EUA Futures Price vs SCC Variations



Notes: The figure shows the deviations of the estimated EUA futures price and the SCC in percentage deviations from their respective steady states.

Appendix E Balanced Growth Path

To carry out our structural parameters estimation via Bayesian estimation, we first need to specify the de-trended economy along its balanced growth path.

The growth rate of Γ_t^Y dictates the economy's growth rate on the balanced growth path.[30] This growth rate is denoted by γ^Y, where:

$$\Gamma_t^Y = \gamma^Y \Gamma_{t-1}^Y \tag{18}$$

Variables that are stationary are represented by lowercase letters, while those that are growing are indicated by uppercase letters. For instance, in the expanding economy, the final firm output, the non-energy output (intermediate variable), and non-energy output are denoted by Y_t, Y_t^{NE} and Y_t^{E}, resepctively. To obtain the de-trended output, one must divide the output in the growing economy by the level of growth progress:

$$y_t = \frac{Y_t}{\Gamma_t^Y} \tag{19}$$

$$y_t^{\mathrm{NE}} = \frac{Y_t^{\mathrm{NE}}}{\Gamma_t^Y} \tag{20}$$

$$\tilde{y}_t^{\mathrm{E}} = \frac{\tilde{Y}_t^{\mathrm{E}}}{\Gamma_t^Y \Gamma_t^E} \tag{21}$$

$$y_t^{\mathrm{E}} = \frac{Y_t^{\mathrm{E}}}{\Gamma_t^Y} \tag{22}$$

where $\Gamma_t^Y \Gamma_t^E = 1$ (given that energy production in EU is not growing over the studied period). In the growing economy, emissions from the energy sector are represented by E_t and are defined as follows:

$$E_t^{\mathrm{E}} = (1 - \mu_t)\epsilon_t^{\varphi_E} \varphi_{\mathrm{E}} Y_t^{\mathrm{E}} \Gamma_t^X \tag{23}$$

$$E_t^{\mathrm{NE}} = \varphi_{\mathrm{NE}} Y_t^{\mathrm{NE}} \Gamma_t^X \tag{24}$$

where Γ_t^X represents the decoupling of CO_2 emissions relative to the output trend. Consequently, in the de-trended economy, the law of motion for CO_2 emissions is expressed

[30] In our setup both final firms and non-energy firms grow at the identical rate Γ_t^Y.

as:

$$e_t^E = (1-\mu_t)\epsilon_t^{\varphi_E}\varphi_{\mathrm{E}} y_t^{\mathrm{E}} \tag{25}$$

$$e_t^{\mathrm{NE}} = \varphi_{\mathrm{NE}} y_t^{\mathrm{NE}} \tag{26}$$

where:

$$e_t^E = \frac{E_t^E}{\Gamma_t^Y \Gamma_t^X} \tag{27}$$

$$e_t^{\mathrm{NE}} = \frac{E_t^{\mathrm{NE}}}{\Gamma_t^Y \Gamma_t^X} \tag{28}$$

The abatement cost in the growing economy is:

$$Z_t = f(\mu_t) Y_t^{\mathrm{E}} \tag{29}$$

In the de-trended economy, the abatement cost is represented as:[31]

$$z_t = f(\mu_t) y_t^{\mathrm{E}} \tag{30}$$

where $z_t = \frac{Z_t}{\Gamma_t^Y}$.

In this context, X_t denotes the cumulative emissions in the atmosphere, while the temperature in the growing economy is represented by T_t^o:

$$X_{t+1} = \eta X_t + E_t^T + E_t^* \tag{31}$$

$$T_{t+1}^o = \zeta_1(\zeta_2 X_t - T_t^o) + T_t^o \tag{32}$$

The de-trended X_t and T_t^o read as follows:

$$\gamma^s x_{t+1} = \eta x_t + e_t^T + e^* \tag{33}$$

$$\gamma^s t_{t+1}^o = \zeta_1(\zeta_2 x_t - t_t^o) + t_t^o \tag{34}$$

[31]Note that μ_t is stationary.

where:

$$x_t = \frac{X_t}{\Gamma_t^Y \Gamma_t^X} \tag{35}$$

$$t_t^o = \frac{T_t^o}{\Gamma_t^Y \Gamma_t^X} \tag{36}$$

with $\gamma^s = \gamma^y \gamma^x$.

In the growing economy, given the aforementioned growth progress, the production functions for both the energy and non-energy sectors are defined as follows:

$$\tilde{Y}_t^{\mathrm{E}} = \varepsilon_t^{A^{\mathrm{E}}} A_t^{\mathrm{E}} (K_t^{\mathrm{E}})^{\alpha_{\mathrm{E}}} (\Gamma_t^Y l_t^{\mathrm{E}})^{1-\alpha_{\mathrm{E}}} \Gamma_t^E \tag{37}$$

$$Y_t^{\mathrm{NE}} = \varepsilon_t^{A^{\mathrm{NE}}} A_t^{\mathrm{NE}} (K_t^{\mathrm{NE}})^{\alpha_{\mathrm{E}}} (\Gamma_t^Y l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{38}$$

$$Y_t = \left((1-\chi)^{\frac{1}{\sigma}} (Y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} Y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{39}$$

where, for both energy and non-energy labor $l_t^{\mathrm{E}}, l_t^{\mathrm{NE}}$, the technology shocks $\varepsilon_t^{A^{\mathrm{E}}}, \varepsilon_t^{A^{\mathrm{NE}}}$ as well as the TFP levels A_t^{E} and A_t^{NE}, are all stationary variables.

De-trending the production functions gives the following:

$$\tilde{y}_t^{\mathrm{E}} = \varepsilon_t^{A^{\mathrm{E}}} A_t^{\mathrm{E}} (k_t^{\mathrm{E}})^{\alpha_{\mathrm{E}}} (l_t^{\mathrm{E}})^{1-\alpha_{\mathrm{E}}} \tag{40}$$

$$y_t^{\mathrm{NE}} = \varepsilon_t^{A^{\mathrm{NE}}} A_t^{\mathrm{NE}} (k_t^{\mathrm{NE}})^{\alpha_{\mathrm{E}}} (l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{41}$$

$$y_t = \left((1-\chi)^{\frac{1}{\sigma}} (y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} (y_t^{\mathrm{E}})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{42}$$

The capital-accumulation equations for both the energy and non-energy sectors in the growing economy read as:

$$K_{t+1}^{\mathrm{E}} = (1-\delta) K_t^{\mathrm{E}} + I_t^{\mathrm{E}} \tag{43}$$

$$K_{t+1}^{\mathrm{NE}} = (1-\delta) K_t^{\mathrm{NE}} + I_t^{\mathrm{NE}} \tag{44}$$

In the de-trended economy, we thus have:

$$\gamma^y k_{t+1}^{\mathrm{E}} = (1-\delta) k_t^{\mathrm{E}} + i_t^{\mathrm{E}} \tag{45}$$

$$\gamma^y k_{t+1}^{\mathrm{NE}} = (1-\delta) k_t^{\mathrm{NE}} + i_t^{\mathrm{NE}} \tag{46}$$

with both capital and investment de-trended variables reading as: $k_t^{\mathrm{NE}} = \frac{K_t^{\mathrm{NE}}}{\Gamma_t^y}$ and $i_t^{\mathrm{NE}} = \frac{I_t^{\mathrm{NE}}}{\Gamma_t^y}$, respectively. The same applies for the energy sector.

Moreover, the economy's resource constraint is:

$$y_t = c_t + g_t + f(\mu_t) y_t^{\mathrm{E}} + i_t^{\mathrm{E}} + i_t^{\mathrm{NE}} \tag{47}$$

Finally, in the growing economy, the utility function is as follow:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1} - \Theta_t^T T_t^o)^{1-\sigma}}{1-\sigma} \right) \tag{48}$$

where C_t is consumption, β the subjective discount factor, and σ the curvature parameter. The de-trended utility function takes the following form:

$$\sum_{t=0}^{\infty} \tilde{\beta}^t \left(\frac{(c_t - \tilde{h} c_{t-1} - \Theta^T t_t^o)^{1-\sigma}}{1-\sigma} \right) \tag{49}$$

where we denote $\tilde{\beta} = \beta(\gamma^y)^{1-\sigma}$, $\tilde{h} = h(\gamma^y)^{-1}$, and $\Theta^T = \Gamma_t^X \Theta_t^T$.

Appendix F The Social Planner Equilibrium: Centralized Economy

The social planner's optimal allocation and plan would aim to maximize the welfare of the society. This is achieved by selecting a sequence of allocations, given the initial conditions for the endogenous state variables, that adhere to the economy's constraints. This equilibrium serves as a benchmark solution, which we use to compare with the allocation derived in the decentralized economy for the carbon policy.

The planners' problem reads as follows:

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \Bigg(& \frac{(c_t - \tilde{h}c_{t-1} - \Theta^T t_t^o)^{1-\sigma^U}}{1-\sigma^U} \\
& + \lambda_t (y_t - c_t - i_t^E - i_t^{\text{NE}} - g_t - f(\mu_t) y_t^E) \\
& + \lambda_t q_t^{\text{NE}} ((1-\delta) k_t^{\text{NE}} + i_t^{\text{NE}} - \gamma^y k_{t+1}^{\text{NE}}) \\
& + \lambda_t q_t^{\text{E}} ((1-\delta) k_t^{\text{E}} + i_t^{\text{E}} - \gamma^y k_{t+1}^{\text{E}}) \\
& + \lambda_t \Psi_t^{\text{NE}} (\varepsilon_t^{A_{\text{NE}}} A^{\text{NE}} (k_t^{\text{NE}})^{\alpha_{\text{NE}}} (l_t^{\text{NE}})^{1-\alpha_{\text{NE}}} - y_t^{\text{NE}}) \\
& + \lambda_t \Psi_t^{\text{E}} (\varepsilon_t^{A_{\text{E}}} A^{\text{E}} (k_t^{\text{E}})^{\alpha_{\text{E}}} (l_t^{\text{E}})^{1-\alpha_{\text{E}}} - y_t^{\text{E}}) \\
& + \lambda_t \Psi_t (((1-\chi)^{\frac{1}{\sigma}} (y_t^{\text{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} y_t^{\text{E}\,\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} - y_t) \\
& + \lambda_t V_t^X (\gamma^s x_{t+1} - \eta x_t - e_t^E - e_t^{\text{NE}} - e^*) \\
& + \lambda_t V_t^T (\gamma^s t_{t+1}^o - \upsilon_1^o (\upsilon_2^o x_t - t_t^o) - t_t^o) \\
& + \lambda_t V_t^{E^{\text{E}}} (e_t^{\text{E}} - (1-\mu_t) \epsilon_t^{\varphi_E} \varphi_{\text{E}} y_t^{\text{E}} \\
& + \lambda_t V_t^{E^{\text{NE}}} (e_t^{\text{NE}} - \varphi_{\text{NE}} y_t^{\text{NE}}) \Bigg)
\end{aligned}$$

where, as we will demonstrate below, the Social Cost of Carbon SCC_t represents the shadow value associated with the temperature damages $\S_T^t$.

The first order conditions (FOCs) that determine SCC_t are the FOC with respect to CO_2 energy emissions e_t^E. In combination with the FOCs with respect to t_t^o and x_t we can pin down the expression of the SCC. Meanwhile, the FOC with respect to μ_t determine the required level of abatement:

$$V_t^{E^E} = V_t^X \tag{50}$$

$$\gamma^s V_t^T = \tilde{\beta} E_t \{ \Lambda_{t+1} \left((1-\zeta_1) V_{t+1}^T + \Theta^T \right) \} \tag{51}$$

$$\gamma^s V_t^X = \tilde{\beta} E_t \{ \Lambda_{t+1} (\zeta_1 \zeta_2 V_{t+1}^T + \eta V_{t+1}^X) \tag{52}$$

$$f'(\mu_t) = \epsilon_t^{\varphi_E} \varphi_{\text{E}} V_t^{E^{\text{E}}} \tag{53}$$

The remaining of the FOCs will be presented in the decentralized economy.

Appendix G The Decentralized Economy

G.1 Households

Households maximize utility over consumption subject to their budget constraint. They choose consumption expenditures and holdings of government bonds,pay taxes and receive dividends for firms they own.

$$\max_{\{c_t, b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{(c_t - \tilde{h}c_{t-1} - \Theta^T t_t^o)^{1-\sigma^U}}{1-\sigma^U}$$

s.t.

$$w_t^{\mathrm{NE}} l_t^{\mathrm{NE}} + w_t^{\mathrm{E}} l_t^{\mathrm{E}} + r_t b_t + \Pi_t^E + \Pi_t^F - t_t = c_t + b_{t+1}$$

From the FOCs, we get:

$$\lambda_t = \varepsilon_t^B \left(c_t - \tilde{h}c_{t-1} - \Theta^T t_t^o\right)^{-\sigma^U} - \varepsilon_{t+1}^B \tilde{\beta}\tilde{h} \left(c_{t+1} - \tilde{h}c_t - \Theta^T t_{t+1}^o\right)^{-\sigma^U} \tag{54}$$

$$\tilde{\beta} r_{t+1} \Lambda_{t+1} = 1 \tag{55}$$

where we note $\Lambda_t = \frac{\lambda_t}{\lambda_{t-1}}$.

G.2 Energy Firms

Energy producers maximize profits by choosing capital investment and labour wages, as well as the investment in abatement as the regulator imposes a carbon price on their level of emissions. The production technology is a Cobb-Douglas, while the abatement investment is a convex function on abatement levels. Capital depreciates and follows a standard law of motion.

The firms' problem reads:

$$\max_{\{y_t^{\mathrm{E}}, i_t^{\mathrm{E}}, \mu_t\}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \Lambda_{t+1} \Pi_t^{\mathrm{E}}$$

where

$$\Pi_t^{\mathrm{E}} = \varepsilon_t^p p_t^{\mathrm{E}} y_t^{\mathrm{E}} - w_t^{\mathrm{E}} l_t^{\mathrm{E}} - i_t^{\mathrm{E}} - f(\mu_t)\, y_t^{\mathrm{E}} - e_t^E \tau_t$$

s.t.

$$y_t^{\mathrm{E}} = \varepsilon_t^{A_{\mathrm{E}}} A^{\mathrm{E}} (k_t^{\mathrm{E}})^{\alpha_{\mathrm{E}}} (l_t^{\mathrm{E}})^{1-\alpha_{\mathrm{E}}}$$
$$e_t^E = (1-\mu_t)\epsilon_t^{\varphi_E} \varphi_{\mathrm{E}} y_t^{\mathrm{E}}$$
$$\gamma^y k_{t+1}^{\mathrm{E}} = (1-\delta) k_t^{\mathrm{E}} + i_t^{\mathrm{E}}$$

The FOCs with respect to capital, investment, labour, abatement, and energy output read as:

$$q_t^{\mathrm{E}} \gamma^y = \tilde{\beta} \Lambda_{t+1} q_{t+1}^{\mathrm{E}} \left((1-\delta) + \alpha_{\mathrm{E}} \Psi_{t+1}^{\mathrm{E}} \frac{y_{t+1}^{\mathrm{E}}}{k_{t+1}^{\mathrm{E}}} \right) \tag{56}$$
$$q_t^{\mathrm{E}} = 1 \tag{57}$$
$$w_t^{\mathrm{E}} = (1-\alpha_{\mathrm{E}}) \frac{y_t^{\mathrm{E}}}{l_t^{\mathrm{E}}} \tag{58}$$
$$f'(\mu_t) = \epsilon_t^{\varphi_E} \varphi_{\mathrm{E}} \tau_t \tag{59}$$
$$\Psi_t^{\mathrm{E}} = \epsilon_t^p p_t^{\mathrm{E}} - (\theta_1 \mu_t^{\theta_2} + \tau_t (1-\mu_t) \epsilon_t^{\varphi_E} \varphi_{\mathrm{E}}) \tag{60}$$

where we denote Ψ_t^{E} and q_t^n the Lagrange multipliers associated with production inputs and investment.

G.3 Non-energy final firms

Non-energy producers maximize profits:

$$\Pi_t^F = y_t - w_t^{\mathrm{NE}} l_t^{\mathrm{NE}} - i_t^{\mathrm{NE}} - \varepsilon_t^p p_t^{\mathrm{E}} y_t^{\mathrm{E}}.$$

s.t.

$$y_t = \left((1-\chi)^{\frac{1}{\sigma}} (y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{61}$$
$$y_t^{\mathrm{NE}} = \varepsilon_t^{A_{\mathrm{NE}}} A^{\mathrm{NE}} (k_t^{\mathrm{NE}})^{\alpha_{\mathrm{NE}}} (l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{62}$$
$$\gamma^y k_{t+1}^{\mathrm{NE}} = (1-\delta) k_t^{\mathrm{NE}} + i_t^{\mathrm{NE}} \tag{63}$$

The FOCs with respect to capital, investment, labour, energy, and non-energy production, yield the factor prices:

$$q_t^{\mathrm{NE}}\gamma^{\mathrm{NE}} = \tilde{\beta}\Lambda_{t+1}q_{t+1}^{\mathrm{NE}}\left((1-\delta) + \Psi_{t+1}^{\mathrm{NE}}\frac{\partial y_{t+1}^{\mathrm{NE}}}{\partial k_{t+1}^{\mathrm{NE}}}\right) \tag{64}$$

$$q_t^{\mathrm{NE}} = 1 \tag{65}$$

$$w_t^{\mathrm{NE}} = \Psi_t^{\mathrm{NE}}\frac{\partial y_{t+1}^{\mathrm{NE}}}{\partial l_{t+1}^{\mathrm{NE}}} \tag{66}$$

$$\epsilon_t^p p_t^E = \Psi_t^{\mathrm{NE}}\frac{\partial y_{t+1}^{\mathrm{NE}}}{\partial y_{t+1}^{\mathrm{E}}} \tag{67}$$

$$q_t^{\mathrm{NE}} = q_t\frac{\partial y_t}{\partial y_t^{\mathrm{NE}}} \tag{68}$$

where we denote Ψ_t^{NE}, and q_t^{NE}, and q_t the Lagrange multipliers associated with production inputs, non-energy investment, and total output, respectively.

We can also easily check that $\Psi_t^{\mathrm{NE}} = 1$ as we are in an RBC case.

G.4 Environmental Policy

When the environmental regulator optimally sets the environmental policy, the carbon price is set equal to the social cost of carbon, as demonstrated in the social planner's case:

$$\tau_t = V_t^{E^E} \tag{69}$$

Alternatively, the regulator might choose to set an emission cap as follows:

$$e_t^E = \bar{Q}\epsilon_t^S \tag{70}$$